

1. Find the slope of the line tangent to the curve $x^3 + 2y^3 - x^2y^4 = x$ at the point $(2, -1)$.

Solution: The slope is $\frac{dy}{dx}$, which we find by implicit differentiation. We get

$$3x^2 + 6y^2y' - 4x^2y^3y' - 2xy^4 = 1,$$

which we solve to get

$$y' = \frac{1 + 2xy^4 - 3x^2}{6y^2 - 4x^2y^3}.$$

At the point $x = 2$, $y = -1$, we have

$$\begin{aligned} y' &= \frac{1 + 4 - 12}{6 + 16} \\ &= -\frac{7}{22}. \end{aligned}$$

2. Find the slope of the line tangent to the curve $y^3 - 2xy^2 + x^4 = 1$ at the point $(1, 2)$.

Solution: To find $\frac{dy}{dx}$, we use implicit differentiation. We get

$$3y^2y' - 4xyy' - 2y^2 + 4x^3 = 0.$$

We solve for y' , getting

$$y' = \frac{2y^2 - 4x^3}{3y^2 - 4xy}.$$

Evaluating this at the point $(1, 2)$, we get

$$\begin{aligned} y' &= \frac{4}{4} \\ &= 1. \end{aligned}$$