

1. Let f be the function given by $f(x) = \sqrt{3x - 6}$.

(a) Find a formula for $f^{-1}(x)$.

Solution: We write $y = \sqrt{3x - 6}$ and solve for x . We get

$$\begin{aligned}y^2 &= 3x - 6 \\y^2 + 6 &= 3x \\x &= \frac{y^2 + 6}{3}\end{aligned}$$

(b) Find the domain and range of f^{-1} .

Solution: Since the domain of f is $[2, \infty)$, the range of f^{-1} must also be $[2, \infty)$. The domain of f^{-1} must be the same as the range of f , which is $[0, \infty)$.

2. Let $f(x) = 3x^2 + 1$, for $x \leq 0$.

(a) Find a formula for $f^{-1}(x)$.

Solution: We write

$$y = 3x^2 + 1, \quad x \leq 0,$$

and solve for x . We get

$$\begin{aligned}y - 1 &= 3x^2, \quad x \leq 0 \\x &= \pm \sqrt{\frac{y - 1}{3}}, \quad x \leq 0, \text{ so} \\x &= -\sqrt{\frac{y - 1}{3}}.\end{aligned}$$

Interchanging x and y , we get

$$\begin{aligned}y &= -\sqrt{\frac{x - 1}{3}} \\f^{-1}(x) &= -\sqrt{\frac{x - 1}{3}}.\end{aligned}$$

(b) Find the domain and range of $f^{-1}(x)$.

Solution: The domain of f^{-1} is the same as the range of f , which is $[1, \infty)$. The range of f^{-1} is the same as the domain of f , which is $(-\infty, 0]$.