

1. A cylindrical can is to have a volume of  $400 \text{ cm}^3$ . Find the dimensions (height and radius) of the can so as to minimize its total surface area. (The surface area comprises the top and bottom and the lateral surface.)
2. One corner of a rectangle is at the origin in the  $xy$ -plane, and the opposite corner lies in the first quadrant, along the line  $y = 7 - \frac{2}{3}x$ . The sides of the rectangle are parallel to the coordinate axes. Find the largest possible area of such a rectangle.
3. Find the positive number  $x$  for which  $5x + \frac{1}{x^2}$  is as small as possible.
4. Find the points on the curve  $y = x^2 - 1$  that are closest to the origin.  
(HINT: Let  $(x, y)$  be a point on the curve, and minimize the square of the distance from  $(0, 0)$  to  $(x, y)$ .)
5. A rectangular garden with an area of 1000 square meters is to be laid out beside a straight river. The bank of the river provides one side of the garden; along the other three sides we need to put up fencing. Find the minimum amount of fencing needed to enclose the garden.