Math 101(04) Solution
Quiz 4 1 November 2001

1. Use differentials or local linearization to estimate $\sqrt[3]{998}$. (Make use of the fact that $\sqrt[3]{1000} = 10$.)

   Solution: Let $f(x) = x^{\frac{1}{3}}$ so that $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$. We find that

$$f'(1000) = \frac{1}{300}$$

   so at $x = 1000$, we get

$$df = \frac{1}{300}\, dx.$$

   Thus $f(998)$ is approximately $f(1000) - \frac{2}{300}$. Since $f(1000) = 10$ we get

$$\begin{aligned} f(998) &\approx 10 - \frac{2}{300} \\ &= \frac{1499}{150}. \end{aligned}$$

2. Sketch the graph of a function f that is continuous on $[0, 4]$, has an absolute maximum at 1, a critical point at 3, an absolute minimum at 4, and no local minimum.