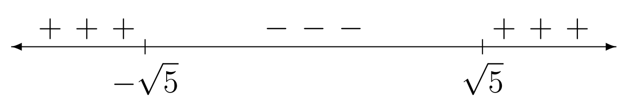
Math 101(04) Solution
Quiz 5 15 November 2001

1. Let $f(x) = x^3 - 15x + 2$. On what intervals is f increasing, and on what intervals is f decreasing? (The domain of f is all real numbers.)

   Solution: We have

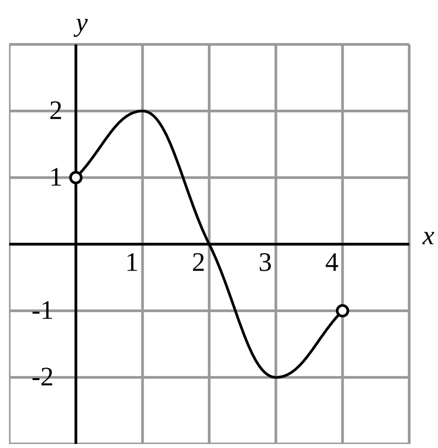
$$\begin{aligned} f'(x) &= 3x^2 - 15 \\ &= 3(x^2 - 5). \end{aligned}$$

   Thus $f'(x) = 0$ when $x = \pm 5$. We construct a sign chart for $f'(x)$



   from which we deduce that f is increasing on $(-\infty, -\sqrt{5}]$ and on $[\sqrt{5}, \infty)$ and decreasing on $[-\sqrt{5}, \sqrt{5}]$.

2. The function g has domain $[0, 4]$ and is differentiable on $(0, 4)$. The graph of the derivative, g', is shown below. Determine the intervals on which the graph of g (the original function) is concave upward and the intervals on which it is concave downward.



   Solution: The graph of g is concave upward on the intervals where g' is increasing, and concave downward on the intervals where g' is decreasing. On the graph, it appears that g' is increasing on $(0, 1]$ and $[3, 4)$ and decreasing on $[1, 3]$, so we conclude that the graph of g will be

   - concave upward on $(0, 1)$ and $(3, 4)$, and
   - concave downward on $(1, 3)$.

   (We leave these intervals open because Stewart's concavity test (p. 243) requires $g''(x) > 0$ or $g''(x) < 0$, and it appears that $g''(x) = 0$ at $x = 1$ and $x = 3$.)