Math 101(04) Solution
Quiz 6 7 December 2001

1. Use four subintervals and left endpoints to estimate the area under the curve $y = \dfrac{1}{1+x^2}$ between $x = 0$ and $x = 2$. You may leave your answer as a sum of fractions.

   Solution: The width of the subintervals is $\dfrac{1}{2}$, and the left endpoints are 0, $\dfrac{1}{2}$, 1, and $\dfrac{3}{2}$. Thus the area is estimated by

$$\begin{aligned} L_4 &= \frac{1}{2}\left[\frac{1}{1+0^2} + \frac{1}{1+\frac{1}{2}^2} + \frac{1}{1+1^2} + \frac{1}{1+\frac{3}{2}^2}\right] \\ &= \frac{1}{2}\left[1 + \frac{4}{5} + \frac{1}{2} + \frac{4}{13}\right] \\ &= \frac{1}{2} \cdot \frac{130+104+65+40}{130} \\ &= \frac{339}{260}. \end{aligned}$$

2. The equation $x^2 = x + 1$ has two solutions, one of them negative and one positive. The positive solution, which is a little more than $3/2$, is an irrational number called the *golden mean*. Find a rational estimate for the golden mean by using Newton's method to approximate a solution to $x^2 = x + 1$. Take $x_1 = \dfrac{3}{2}$ and find x_2. Write your answer as a fraction, *not* a decimal.

   Solution: We seek a solution to $x^2 - x - 1 = 0$. The iteration rule for Newton will be

$$x_{n+1} = x_n - \frac{x_n^2 - x_n - 1}{2x_n - 1}.$$

   Using $x_1 = \dfrac{3}{2}$, we get

$$\begin{aligned} x_2 &= \frac{3}{2} - \frac{\left(\frac{3}{2}\right)^2 - \frac{3}{2} - 1}{2\left(\frac{3}{2}\right) - 1} \\ &= \frac{3}{2} - \frac{\frac{9}{4} - \frac{3}{2} - 1}{2} \\ &= \frac{3}{2} - \frac{-\frac{1}{4}}{2} \\ &= \frac{3}{2} + \frac{1}{8} \\ &= \frac{26}{16} = \frac{13}{8}. \end{aligned}$$