Math 203(01) **Practice Exam I** **5 October 2001**

1. Let $\vec{v} = 3\vec{\imath} - 4\vec{\jmath} + \vec{k}$ and $\vec{w} = \vec{\imath} - 2\vec{\jmath} + 2\vec{k}$.

   (a) Find the cosine of the angle θ between $\vec{v}$ and $\vec{w}$.

   (b) If $\vec{s} = 2\vec{\imath} + 3\vec{\jmath} + \alpha\vec{k}$, and $\vec{s}$ is perpendicular to $\vec{w}$, find the value of α.

2. Let A be the point $(1, 2, 1)$, B be the point $(3, 2, 5)$ and C be the point $(0, 4, 5)$.

   (a) Find an equation for the plane containing the points A, B, and C.

   (b) Find the area of triangle ABC.

3. A pilot, flying at an altitude of 3000 feet, steers her airplane on a heading of due north at an airspeed of 100 knots. The ground track of the airplane is in a direction 6° east of north, and the airplane's ground speed is 80 knots. How fast is the wind blowing at 3000 feet?

   (Give the answer both in exact form and as a decimal approximation.)

4. Let $\vec{w} = 4\vec{\imath} - \vec{\jmath} + 3\vec{k}$.

   (a) Suppose $\vec{w} = \vec{w}_{\text{parallel}} + \vec{w}_{\text{perp}}$, where $\vec{w}_{\text{parallel}}$ is parallel to the vector $\vec{\imath} + \vec{\jmath} + \vec{k}$ and $\vec{w}_{\text{perp}}$ is perpendicular to the vector $\vec{\imath} + \vec{\jmath} + \vec{k}$.
   Write $\vec{w}_{\text{parallel}}$ and $\vec{w}_{\text{perp}}$ in components.

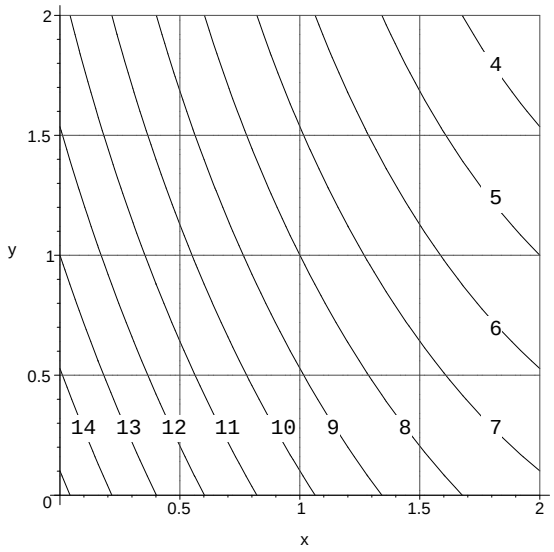
   (b) What is the distance from the point $(4, -1, 3)$ to the line $x = y = z$?

5. Let $f(x, y) = \dfrac{y + \cos y}{1 + x^2 + y^2}$.

(a) Find the local linearization of f at $(2, 0)$.

(b) Suppose that x increases from 2 to 2.1. By what amount would y have to change (from 0) in order to keep $f(x, y)$ approximately constant?

6. Here is a contour plot of a function $g(x, y)$.



Using the contour plot to approximate whatever values you need, construct an approximate local linearization for g at the point $(1, 1)$.

7. Let $f(x, y, z) = x^2 - y^2 + \dfrac{z^2}{4}$.

(a) Sketch the level surface $f(x, y, z) = 6$. Identify all the points where the surface intersects the coordinate axes.

(b) Find an equation for the plane tangent to the surface $f(x, y, z) = 6$ at the point $(3, 2, 2)$.

8. Let $g(x, y) = x^2 y - x \ln y$.

   (a) Let $\vec{u} = \dfrac{1}{13}(5\vec{\imath} - 12\vec{\jmath})$. Find $g_{\vec{u}}(3, 2)$.

   (b) Find a vector pointing in the direction in which g decreases most rapidly from the point $(3, 2)$.