**Practice Problems after Hour Exam 2**

1. Compute
$$\int_0^1 \int_{y^2}^1 e^{x^{\frac{3}{2}}}\, dx\, dy.$$

2. Let $C$ be the helix centered on the $z$-axis, circling counterclockwise, making two full rotations while rising from $(4, 0, 0)$ to $(4, 0, 8)$.
   (a) Parametrize $C$.
   (b) Let $\vec{F} = -y\vec{\imath} + x\vec{\jmath} - z\vec{k}$. Compute $\int_C \vec{F} \cdot d\vec{r}$.
   (c) Let $\vec{G} = 2xy^2z^2\vec{\imath} + 2yx^2z^2\vec{\jmath} + (2zx^2y^2 + 1)\vec{k}$. Find $\int_C \vec{F} \cdot d\vec{r}$.

3. Let $S$ be the surface parametrized by
$$\vec{r}(s, t) \quad = \quad s \cos t\vec{\imath} + s \sin t\vec{\jmath} + \sin s\vec{k}$$
   with $\pi \le s \le 2\pi$ and $0 \le t \le 2\pi$.
   (a) Sketch $S$ and describe the orientation given by its parametrization.
   (b) Set up, but do not evaluate, an integral for the surface area of $S$.
   (c) Let $\vec{F} = y\vec{\imath} - x\vec{\jmath} + \vec{k}$. Calculate the flux of $\vec{F}$ upward through $S$.

4. Let $C_1$ be the parabolic curve $z = y^2$, $x = 0$, $-2 \le y \le 2$, oriented in the positive $y$ direction. Let $C_2$ be the line segment from $(0, 2, 4)$ to $(0, -2, 4)$. Let $C$ be the union of $C_1$ and $C_2$, so that $C$ is a closed curve.
   Let $\vec{F} = \cos(x)\vec{\imath} + 3z\vec{\jmath} - y\vec{k}$.
   (a) Compute $\oint_C \vec{F} \cdot d\vec{r}$ by parametrizing $C_1$ and $C_2$. (You will have two integrals.)
   (b) Compute curl $\vec{F}$ and use Stokes's theorem to verify your answer to part (4a).

5. Let $W$ be the solid region $x^2 + y^2 \le 4$ and $0 \le z \le 5$. Let
$$\vec{F}(x, y, z) \quad = \quad (2x + y \cos(z))\vec{\imath} + (3y + x \sin(z))\vec{\jmath} + z(5 - z)\vec{k}.$$

(a) Explain why the flux of $\vec{F}$ through the top and bottom faces of $W$ is zero.

(b) Use the divergence theorem to compute the total flux of $\vec{F}$ through the boundary of $W$.

(c) Determine the flux of $\vec{F}$ outward through the lateral surface of the boundary of $W$ (that is, the part of the surface that isn't the top or the bottom).