Math 203(01) Solution
Quiz 10 7 December 2001

1. Let C be the quarter-circle parametrized by

$$\vec{r}(t) = 2\cos(t)\vec{\imath} + 2\sin(t)\vec{\jmath}, \qquad 0 \le t \le \frac{\pi}{2}.$$

Let $\vec{F} = -2y\vec{\imath} + x\vec{\jmath}$. Compute $\int_C \vec{F}\cdot d\vec{r}$.

Solution: We have

$$\vec{r}'(t) = -2\sin(t)\vec{\imath} + 2\cos(t)\vec{\jmath}$$

and

$$\vec{F}(\vec{r}(t)) = = -4\sin(t)\vec{\imath} + 2\cos(t)\vec{\jmath}$$

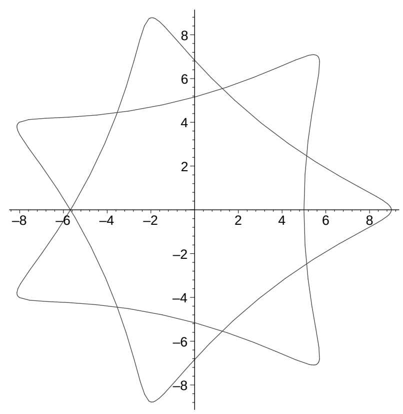
so that

$$\begin{aligned}
\int_C \vec{F}(\vec{r}(t))\cdot d\vec{r} &= \int_0^{\frac{\pi}{2}} 8\sin^2(t) + 4\cos^2(t)\,dt \\
&= \int_0^{\frac{\pi}{2}} 4 + 4\sin^2(t)\,dt \\
&= \frac{4\pi}{2} + 4\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2t}{2}\right) dt \\
&= 2\pi + 2\left[t - \sin 2t\right]_0^{\frac{\pi}{2}} \\
&= 2\pi + \pi = 3\pi.
\end{aligned}$$

2. At right is a *Maple* drawing of the curve C parametrized by

$$\begin{aligned}
\vec{r}(t) &= (7\cos(t) - 2\cos(5t/2))\vec{\imath} \\
&\quad + (7\sin(t) - 2\sin(5t/2))\vec{\jmath},
\end{aligned}$$

$0 \le t \le 4\pi$.



Let

$$\vec{F} = 3x^2e^y\vec{\imath} + (x^3e^y + \cos y)\vec{\jmath}.$$

Find $\oint_C \vec{F}\cdot d\vec{r}$.

Solution: Let $f(x,y) = x^3e^y + \sin y$. Then

$$\begin{aligned}
\nabla f &= 3x^2e^y\vec{\imath} + (x^3e^y + \cos y)\vec{\jmath} \\
&= \vec{F}
\end{aligned}$$

so $\vec{F}$ is a gradient field. Since C is a closed curve, we conclude that $\oint_C \vec{F}\cdot d\vec{r} = 0$.