Math 203 (Quenell) Solution
Quiz 2 20 September 2001

1. Let $\vec{v} = 3\vec{\imath} - 2\vec{\jmath} + \vec{k}$ and $\vec{w} = \vec{\imath} - 5\vec{\jmath} + 2\vec{k}$. Find the cosine of the angle θ between $\vec{v}$ and $\vec{w}$.

   Solution: We know $\cos\theta = \dfrac{\vec{v}\cdot\vec{w}}{||\vec{v}||\,||\vec{w}||}$. We find

$$\begin{aligned} ||\vec{v}|| &= \sqrt{14} \\ ||\vec{w}|| &= \sqrt{30} \\ \vec{v}\cdot\vec{w} &= 3 + 10 + 2 \\ &= 15, \end{aligned}$$

   so we conclude that $\cos\theta = \dfrac{15}{\sqrt{420}}$.

2. Let $\vec{v} = 5\vec{\imath}$. Let $\vec{w} = 2\vec{\imath} + 2\vec{\jmath} - \vec{k}$. Write $\vec{v}$ as the sum of two vectors, one parallel to $\vec{w}$ and one perpendicular to $\vec{w}$.

   Solution: We first find $\vec{u}$, a unit vector in the direction of $\vec{w}$. Since $||\vec{w}|| = 3$, we have

$$\vec{u} = \frac{2}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} - \frac{1}{3}\vec{k}.$$

   Next, the length of the component of $\vec{v}$ parallel to $\vec{u}$ is given by

$$\vec{v}\cdot\vec{u} = \frac{10}{3}.$$

   We get

$$\begin{aligned} \vec{v}_{\text{parallel}} &= \frac{10}{3}\vec{u} \\ &= \frac{20}{9}\vec{\imath} + \frac{20}{9}\vec{\jmath} - \frac{10}{9}\vec{k} \\ \vec{v}_{\text{perp}} &= \vec{v} - \vec{v}_{\text{parallel}} \\ &= \frac{25}{9}\vec{\imath} - \frac{20}{9}\vec{\jmath} + \frac{10}{9}\vec{k}. \end{aligned}$$

3. Find the area of the triangle whose vertices are the points $(0, 0, 0)$, $(3, 2, 5)$, and $(-2, 4, -1)$.

   Solution: The are of this triangle is half the length of the cross product of the vectors $vecv = 3\vec{\imath} + 2\vec{\jmath} + 5\vec{k}$ and $\vec{w} = -2\vec{\imath} + 4\vec{\jmath} - \vec{k}$. We get

$$\begin{aligned} \vec{v}\times\vec{w} &= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & 2 & 5 \\ -2 & 4 & -1 \end{vmatrix} \\ &= (-2 - 20)\vec{\imath} - (-3 + 10)\vec{\jmath} + (12 + 4)\vec{k} \\ &= -22\vec{\imath} - 7\vec{\jmath} + 16\vec{k}. \end{aligned}$$

   Thus the area of the triangle is

$$\begin{aligned} \frac{1}{2}||\vec{v}\times\vec{w}|| &= \frac{1}{2}\sqrt{484 + 49 + 256} \\ &= \frac{\sqrt{789}}{2}. \end{aligned}$$