Math 203 (Quenell) Solution
Quiz 5 18 October 2001

1. Given $z = xe^{-(x^2+y^2)}$ with $x = r\cos\theta$ and $y = r\sin\theta$, find $\dfrac{\partial z}{\partial r}$. Be sure to express your answer in terms of r and θ.

   Solution: We have

$$\begin{aligned}\frac{\partial z}{\partial x} &= e^{-(x^2+y^2)} - 2x^2e^{-(x^2+y^2)}\\ \frac{\partial z}{\partial y} &= -2xye^{-(x^2+y^2)}\end{aligned}$$

   and

$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial y}{\partial r} = \sin\theta.$$

   By the chain rule, then

$$\begin{aligned}\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}\\ &= (1-2x^2)e^{-(x^2+y^2)}\cos\theta - 2xye^{-(x^2+y^2)}\sin\theta\\ &= (1-2r^2\cos^2\theta)e^{-r^2}\cos\theta - 2r^2e^{-r^2}\sin^2\theta\cos\theta\\ &= (1-2r^2\cos^2\theta - 2r^2\sin^2\theta)e^{-r^2}\cos\theta\\ &= (1-2r^2)e^{-r^2}\cos\theta.\end{aligned}$$

2. Let $f(x,y) = x^2e^{2y}$. Find the second partial derivatives of f.

   Solution: We have

$$\frac{\partial f}{\partial x} = 2xe^{2y}; \qquad \frac{\partial f}{\partial y} = 2x^2e^{2y}.$$

   The second partials are

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} &= 2e^{2y}\\ \frac{\partial^2 f}{\partial y\partial x} &= 4xe^{2y}\\ \frac{\partial^2 f}{\partial y^2} &= 4x^2e^{2y}.\end{aligned}$$