Math 203 (Quenell) Solution
Quiz 6 25 October 2001

1. Let $f(x,y) = e^y\sqrt{x}$. Find the quadratic Taylor approximation for f about $(1,0)$.

   Solution: We have

$$\frac{\partial f}{\partial x} = \frac{e^y}{2\sqrt{x}} \quad \text{and} \quad \frac{\partial f}{\partial y} = e^y\sqrt{x}$$

   so that

$$\left.\frac{\partial f}{\partial x}\right|_{(1,0)} = \frac{1}{2} \quad \text{and} \quad \left.\frac{\partial f}{\partial y}\right|_{(1,0)} = 1.$$

   For the second partials, we get

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= -\frac{1}{4}x^{-\frac{3}{2}}e^y \\ \frac{\partial^2 f}{\partial x \partial y} &= \frac{e^y}{2\sqrt{x}} \\ \frac{\partial^2 f}{\partial y^2} &= e^y\sqrt{x}. \end{aligned}$$

   Evaluating these at $(1,0)$, we get

$$\begin{aligned} \left.\frac{\partial^2 f}{\partial x^2}\right|_{(1,0)} &= -\frac{1}{4} \\ \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(1,0)} &= \frac{1}{2} \\ \left.\frac{\partial^2 f}{\partial y^2}\right|_{(1,0)} &= 1. \end{aligned}$$

   The quadratic approximation is given by

$$Q(x,y) = 1 + \frac{1}{2}(x-1) + y - \frac{1}{8}(x-1)^2 + \frac{1}{2}(x-1)y + \frac{1}{2}y^2.$$