Math 203(01) Solution
Quiz 8 8 November 2001

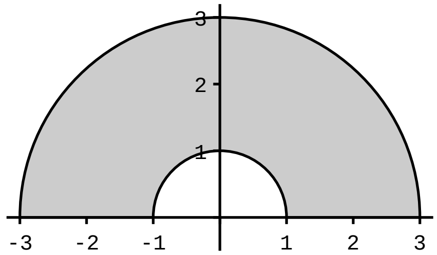
1. Let R be the region in the first quadrant of the xy-plane bounded by the lines $x = 0$ and $y = 4$ and the parabola $y = x^2$. Fill in the limits of integration:

$$\iint_R f(x,y)\,dA$$

$$= \int_{\boxed{0}}^{\boxed{2}} \int_{\boxed{x^2}}^{\boxed{4}} f(x,y)\,dy\,dx$$

$$= \int_{\boxed{0}}^{\boxed{4}} \int_{\boxed{0}}^{\boxed{\sqrt{y}}} f(x,y)\,dx\,dy.$$

2. Let $f(x,y) = \sqrt{x^2+y^2}$. Compute $\iint f(x,y)\,dA$ over the region shown.



Solution: This looks like a job for polar coordinates. The region appears to be bounded by the circles $r = 1$ and $r = 3$ and the rays $\theta = 0$ and $\theta = \pi$. The given function, converted to polar coordinates, becomes $f = r$. We have

$$\begin{aligned}
\iint f\,dA &= \int_0^\pi \int_1^3 r\,r\,dr\,d\theta \\
&= \int_0^\pi \int_1^3 r^2\,dr\,d\theta \\
&= \int_0^\pi \left[\frac{r^3}{3}\right]_1^3 d\theta \\
&= \int_0^\pi \frac{26}{3}\,d\theta \\
&= \frac{26\pi}{3}.
\end{aligned}$$