Math 203(01) Solution
Quiz 9 29 November 2001

1. Write a set of parametric equations for the circle of radius 2, parallel to the xz-plane, centered at the point $(0,4,0)$. Don't forget to include limits on the parameter.

   Solution:

$$\begin{cases} x(t) &= 2\cos t \\ y(t) &= 4 \\ z(t) &= 2\sin t, \end{cases}$$
$$0 \le t \le 2\pi$$

2. An aerobatic aircraft enters a spin at $t = 0$, following the trajectory

$$\vec{r}(t) \quad = \quad 75\cos\left(\frac{2\pi t}{5}\right)\vec{\imath} + 75\sin\left(\frac{2\pi t}{5}\right)\vec{\jmath} - 16t^2\vec{k}$$

   (with t in seconds and distances in feet). Find the acceleration vector at $t = 5$ seconds (that is, after one complete turn).

   Solution: We have

$$\begin{aligned} \vec{r}'(t) &= -30\pi\sin\left(\frac{2\pi t}{5}\right)\vec{\imath} + 30\pi\cos\left(\frac{2\pi t}{5}\right)\vec{\jmath} - 32t\vec{k} \\ \vec{r}''(t) &= -12\pi^2\cos\left(\frac{2\pi t}{5}\right)\vec{\imath} - 12\pi^2\sin\left(\frac{2\pi t}{5}\right)\vec{\jmath} - 32\vec{k} \end{aligned}$$

   Thus $\vec{r}''(5) = -12\pi^2\vec{\imath} - 32\vec{k}$.

3. Sketch the surface parametrized by

$$\begin{aligned} \vec{r}(s,t) &= s\cos(t)\,\vec{\imath} + s\sin(t)\,\vec{\jmath} + (2-s)\vec{k}, \\ & \quad 1 \le s \le 2, \quad 0 \le t \le 2\pi. \end{aligned}$$

   This is part of a cone, opening downward, sitting on the xy-plane. Here is a sketch: