Math 311 Problem Set #1 Due 13 September 2002

Reading: Gallian, Chapters 0, 1, and 2.

Exercises: Write your solutions in complete sentences.

1. (Chapter 1, Exercises 2 and 3) Write out a complete Cayley table for D_3. Be sure to explain your notation for the elements of D_3, using diagrams if necessary. Use your table to show that D_3 is not Abelian.

2. (Chapter 1, Exercise 11) Find elements A, B, and C of D_4 such that $AB = BC$ but $A \neq C$.

3. (Chapter 2, Exercise 3) Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group, but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group. (You may use without proof the fact that modular multiplication is associative.)

4. (Chapter 2, Exercise 14) Let G be a group with the following property: If a, b, and c belong to G and $ab = ca$, then $b = c$. Prove that G is Ablelian.

5. (Part of Chapter 2, Exercise 23) Let G be a finite group. Prove that each element of G appears exactly once in each row of a Cayley table for G.

Cultural aside:

Mathematics consists essentially of:

a) proving the obvious;
b) proving the not so obvious; and
c) proving the obviously untrue.

Mathematicians are allowed to make very heavy weather of showing what everyone already knows. For example, it took mathematicians until the 1800s to prove that $1+1 = 2$, and not before the late 1970s were they confident of proving that any map requires no more than four colours to make it look nice, a fact known by cartographers for centuries.

There are many not-so-obvious things which can be proved true too. Like the fact that for any group of twenty-three people, there's an even chance two or more of them share a birthday. (With groups of twins this becomes almost certain. Not quite certain, as you will of course point out; they might all have been born either side of midnight).

Mathematicians are also fond of proving things which are obviously false, like all straight lines being curved, and an engaged telephone being just as likely to be free if you ring again immediately after, as if you wait twenty minutes. They also like disproving things which are obviously true, for example that the shortest distance between two points on the earth's surface on an airline route always goes across Anchorage, Alaska.

Robert Ainsley, *Bluff Your Way in Maths*