Math 311 Problem Set #11 Due 13 December 2002

Reading: Gallian, Chapter 9.

Exercises: Write your solutions in complete sentences.

1. How many distinct fifteen-bead necklaces can we make using six red beads, six blue beads, and three green beads?

2. How many different ways can we color a dodecahedron using nine black faces and three white faces?

3. (Chapter 9, Exercise 7) Prove that if H has index 2 in G, then H is normal in G.

4. As in the last problem set, write

$$D_n = \{e, r, r^2, \ldots, r^{n-1}, f, rf, r^2f, \ldots, r^{n-1}f\}$$

for $n \geq 3$. Suppose H is a normal subgroup of D_n and $f \in H$. If n is odd, prove that $H = D_n$. What happens if n is even?

Cultural aside:

"This is the fellow we've been waiting for," Chattan says to Robson. "The one we could have used in Algiers."

"Yes!" Robson says. "Welcome to Detachment 2701, Captain Waterhouse."

"2702," Waterhouse says.

Chattan and Robson look every so mildly startled.

"We can't use 2701 because it is the product of two primes."

"I beg your pardon?" Robson says.

One thing Waterhouse likes about these Brits is that when they don't know what the hell you are talking about, they are at least open to the possibility that it might be their fault. Robson has the look of a man who has come up through the ranks. A Yank of that type would already be scornful and blustery.

"Which ones?" Chattan says. That is encouraging; he at least knows what a prime number is.

"73 and 37," Waterhouse says.

This makes a profound impression on Chattan. "Ah, yes, I see." He shakes his head. "I shall have to give the Prof a good chaffing about this."

Robson has cocked his head far to one side so that it is almost resting upon the thick woolly beret chucked into his epaulet. He is squinting, and has an aghast look about him. His hypothetical Yank counterpart would probably demand, at this point, a complete explanation of prime number theory, and when it was finished, denounce it as horseshit.

Neal Stephenson, *Cryptonomicon*