

**Reading:** Gallian, Chapter 4.

**Exercises:** Write your solutions in complete sentences.

1. List all the elements of order 12 in  $\mathbb{Z}_{12000}$ . List all the elements of order 12 in  $\mathbb{Z}_{24000}$ .
2. (Based on Chapter 4, exercise 59)  
If  $|a^5| = 12$ , what are the possibilities for  $|a|$ ?  
If  $|b^4| = 12$ , what are the possibilities for  $|b|$ ?  
If  $|c^4| = 11$ , what are the possibilities for  $|c|$ ?
3. Assume  $a$  is an element of some group  $G$ , and  $|a^6| = K$ , where  $K$  is some positive integer. How many possibilities are there for the order of  $a$ ? Your answer will depend on  $K$ .
4. (Based on Chapter 4, exercise 55) Let  $a$  and  $b$  be elements of a group  $G$  with  $|a| = 24$  and  $|b| = 15$ . Let  $H = |\langle a \rangle \cap \langle b \rangle|$ . Prove that  $H$  is a subgroup of  $G$  and that  $|H|$  is either 1 or 3.
5. (Chapter 4, exercise 64) Suppose  $G$  is a finite group with the property that every non-identity element has prime order. If  $Z(G)$  contains some element other than the identity, prove that every non-identity element of  $G$  has the same order.

### Cultural aside:

“Your hair wants cutting,” said the Hatter. He had been looking at Alice for some time with great curiosity, and this was his first speech.

“You should learn not to make personal remarks,” Alice said with some severity: “It’s very rude.”

The Hatter opened his eyes very wide on hearing this; but all he *said* was, “Why is a raven like a writing-desk?”

“Come, we shall have some fun now!” thought Alice. “I’m glad they’ve begun asking riddles – I believe I can guess that,” she added aloud.

“Do you mean that you think you can find out the answer to it?” said the March Hare.

“Exactly so,” said Alice.

“Then you should say what you mean,” the March Hare went on.

“I do,” Alice hastily replied; “at least – at least I mean what I say – that’s the same thing, you know.”

“Not the same thing a bit!” said the Hatter. “Why, you might just as well say that ‘I see what I eat’ is the same thing as ‘I eat what I see’!”

“You might just as well say,” added the March Hare, “that ‘I like what I get’ is the same thing as ‘I get what I like’!”

“You might just as well say,” added the Dormouse, which seemed to be talking in its sleep, “that ‘I breathe when I sleep’ is the same thing as ‘I sleep when I breathe’!”

“It *is* the same thing with you,” said the Hatter, and here the conversation dropped, and the party sat silent for a minute, while Alice thought over all she could remember about ravens and writing-desks, which wasn’t much.

Lewis Carroll, *Alice’s Adventures in Wonderland*