

Reading: Gallian, Chapter 5.

Exercises: Write your solutions in complete sentences.

1. (Chapter 5, exercise 7) List the possible orders for elements of S_6 , A_6 , S_7 , and A_7 .
2. (Based on Chapter 5, exercise 35) How many elements of order 5 are there in S_6 ? How many elements of order 6 are there in S_6 ?
3. (Chapter 5, exercise 19) Show that if H is a subgroup of S_n , then either every member of H is an even permutation or exactly half of the members are even.
4. (Chapter 5, exercise 26) Show that $(1\ 2\ 3\ 4)$ is not a product of 3-cycles.
5. (Based on Chapter 5, exercise 28) Let $\beta = (1\ 2\ 3)(1\ 4\ 5)$. Write β^{99} in cycle form. Let $\gamma = (1\ 2\ 3\ 4)(2\ 4)(3\ 5)$. Write γ^{99} in cycle form.
6. (Chapter 5, exercise 40) Prove that S_n is non-Abelian for all $n \geq 3$.

Cultural aside:

"You're wasting your time," Doc Daneeka was forced to tell him.

"Can't you ground someone who's crazy?"

"Oh, sure. I have to. There's a rule saying I have to ground anyone who's crazy."

"Then why don't you ground me? I'm crazy. Ask Clevinger."

"Clevinger? Where *is* Clevinger? You find Clevinger and I'll ask him."

"Then ask any of the others. They'll tell you how crazy I am."

"They're crazy."

"Then why don't you ground them?"

"Why don't they ask me to ground them?"

"Because they're crazy, that's why."

"Of course they're crazy," Doc Daneeka replied. "I just told you they're crazy, didn't I? And you can't let crazy people decide whether you're crazy or not, can you?"

Yossarian looked at him soberly and tried another approach. "Is Orr crazy?"

"He sure is," Doc Daneeka said.

"Can you ground him?"

"I sure can. But first he has to ask me to. That's part of the rule."

"Then why doesn't he ask you to?"

"Because he's crazy," Doc Daneeka said. "He has to be crazy to keep flying combat missions after all the close calls he's had. Sure, I can ground Orr. But first he has to ask me to."

"That's all he has to do to be grounded?"

"That's all. Let him ask me."

"And then you can ground him?" Yossarian asked.

"No. Then I can't ground him."

"You mean there's a catch?"

"Sure there's a catch," Doc Daneeka replied. "Catch-22. Anyone who wants to get out of combat duty isn't really crazy."

There was only one catch and that was Catch-22, which specified that a concern for one's own safety in the face of dangers that were real and immediate was the process of a rational mind. Orr was crazy and could be grounded. All he had to do was ask; and as soon as he did, he would no longer be crazy and would have to fly more missions. Orr would be crazy to fly more missions and sane if he didn't but if he was sane he had to fly them. If he flew them he was crazy and didn't have to; but if he didn't want to he was sane and had to. Yossarian was moved very deeply by the absolute simplicity of this clause of Catch-22 and let out a respectful whistle.

"That's some catch, that Catch-22," he observed.

"It's the best there is," Doc Daneeka agreed.

Joseph Heller, *Catch-22*