Math 311 Problem Set #6 Due 25 October 2002

Reading: Gallian, Chapter 6.

Exercises: Write your solutions in complete sentences.

1. (Chapter 6, exercise 7) Show that S_4 is not isomorphic to D_{12}.

2. (Chapter 6, exercise 10) Let G be a group and let $\alpha : G \to G$ by $\alpha(g) = g^{-1}$. Prove that α is an automorphism if and only if G is Abelian.

3. (Chapter 6, exercise 14) Find $\mathrm{Aut}(\mathbb{Z}_6)$. (That is, list all the possible automorphisms of $\mathbb{Z}_6$. Explain how you know each one is really an automorphism, and how you know there aren't any others.)

4. (Chapter 6, Problem 27) Let $\mathbb{C}$ denote the complex numbers and and

$$M = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}.$$

Prove that $\mathbb{C}$ and M are isomorphic under addition. (You may assume that $\mathbb{C}$ and M are groups under the operations of addition and matrix addition, respectively.) Let $\mathbb{C}^*$ denote the non-zero elements of $\mathbb{C}$ with the operation of multiplication and M^* denote the non-zero elements of M with the operation of matrix multiplication. Prove that $\mathbb{C}^*$ and M^* are isomorphic. (You may assume that $\mathbb{C}^*$ and M^* are groups.)

5. Suppose G is a finite Abelian group and G has no element of order 2. Let $\varphi : G \to G$ by

$$\varphi(g) = g^2$$

for $g \in G$. Show that φ is an automorphism of G. Find an example of an infinite Abelian group in which this mapping is not an automorphism.

HINT: First prove this Lemma: For every $g \in G$, the order of g is finite and odd.

Cultural aside:

The author of the bill was a physician, Edwin J. Goodman, M.D., of Solitude, Posey County, Indiana, and it was introduced in the Indiana House on January 18, 1897, by Mr. Taylor I. Record, Represenatative from Posey County. It was entitled "A bill introducing a new Mathematical truth," and it became House Bill No. 246; copies of the bill are preserved in the Archives Division of the Indiana State Library …

The preamble to the bill informs us that this is

> *A bill for an act introducing a new mathematical truth and offered as a contribution to education to be used only by the State of Indiana free of cost by paying any royalties whatever on the same, provided it is accepted an adopted by the official action of the legislature in 1897.*

The bill consisted of three sections. Section 1 starts off like this:

> *Be it enacted by the General Assembly of the State of Indiana: It has been found that the circular area is to the quadrant of the circumference, as the area of an equilateral rectangle is to the square on one side. The diameter employed as the linear unit according to the present rule in computing the circle's area is entirely wrong …*

An "equilateral rectangle" is, of course, a square, so that the first statement does not make any sense at all; but if we give the author the benefit of the doubt and assume that this is a transcript error for "equilateral triangle," then what Mr. Goodwin of Solitude, Posey County, had discovered in his first statement was the equivalent of $\pi = 16/\sqrt{3} = 9.2736\ldots$, which probably represents the biggest overestimate of π in the history of mathematics. …

The bill was, perhaps symbolically, referred to the House Committee on Swamp Lands, which passed it on to the Committee of Education, and the latter reported it back to the House "with recommendation that said bill do pass." On February 5, 1897, the House passed the learned treatise unanimously (67 to 0).

Five days later the bill went to the Senate, where it was referred, for unknown reasons, to the Committee on Temperance. The Committee on Temperance, too, reported it back to the Senate with the recommendation that it pass the bill, and it passed the first reading without comment.

Petr Beckmann, *A History of* π