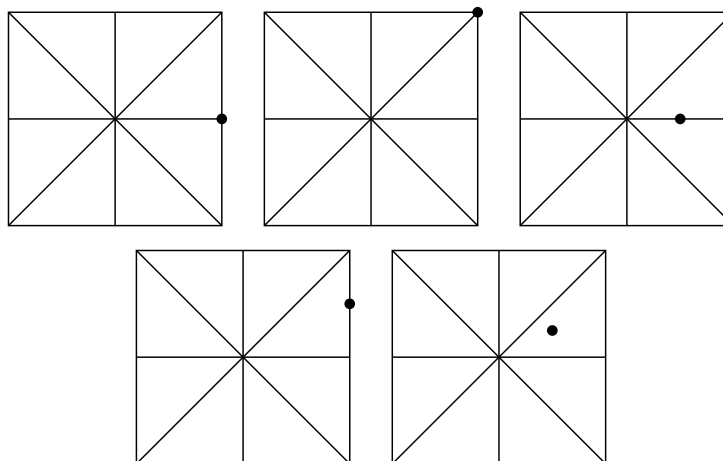


Reading: Gallian, Chapter 8.

Exercises: Write your solutions in complete sentences.

1. (Chapter 7, exercise 24) Let G be a group of order 25. Prove that either G is cyclic or every element $g \in G$ satisfies $g^5 = e$.
2. Suppose G is a group of permutations of a finite set S . Let s_1 and s_2 be elements of S . Prove that either $\text{orb}_G(s_1) = \text{orb}_G(s_2)$ or $\text{orb}_G(s_1) \cap \text{orb}_G(s_2)$ is empty.
3. (Chapter 7, exercise 12) Let G be a group and suppose $|G| = 12$. Prove that G must contain an element of order 2.
4. (Based on Chapter 7, exercise 42) The group D_4 acts as a group of permutations on the points of the square. For each diagram below, locate the points in the orbit of the indicated point under D_4 , and list the elements of the stabilizer of the indicated point. (The axes of symmetry are drawn in only for reference.)



5. Let H be the cyclic subgroup of S_4 generated by $(1\ 2\ 3\ 4)$. Let $a = (1\ 2)$ and $b = (1\ 2\ 3\ 4)$ be elements of S_4 . Draw a Schreier diagram for S_4 modulo H on the generating set $\{a, b\}$.
(HINT: The right cosets are H, Ha, Hab, Hab^2, Hab^3 , and Hab^2a .)

Cultural aside:

The Total Perspective Vortex derives its picture of the whole Universe on the principle of extrapolated matter analyses.

To explain – since every piece of matter in the Universe is in some way affected by every other piece of matter in the Universe, it is in theory possible to extrapolate the whole of creation – every sun, every planet, their orbits, their composition, and their economic and social history from, say, one small piece of fairy cake.

The man who invented the Total Perspective Vortex did so basically in order to annoy his wife.

Trin Tragula – for that was his name – was a dreamer, a thinker, a speculative philosopher or, as his wife would have it, an idiot.

And she would nag him incessantly about the utterly inordinate amount of time he spent staring out into space, or mulling over the mechanics of safety pins, or doing spectrographic analyses of pieces of fairy cake.

“Have some sense of proportion!” she would say, sometimes as often as thirty-eight times in a single day.

And so he built the Total Perspective Vortex – just to show her.

And into one end, he plugged the whole of reality as extrapolated from a piece of fairy cake, and into the other, he plugged his wife: so that when he turned it on she saw in one instant the whole infinity of creation and herself in relation to it.

To Trin Tragula’s horror, the shock completely annihilated her brain, but to his satisfaction he realized that he had proved conclusively that if life is going to exist in a Universe of this size, then one thing it cannot afford to have is a sense of proportion.

Douglas Adams, *The Hitch-Hiker’s Guide to the Galaxy*