

1. A *group* is a set  $G$  together with a binary operation  $\star$ . The set must be closed under the operation, and the operation must satisfy three further properties. List the properties and describe each one.

Solution:

- (a) Associativity: we must have  $(a \star b) \star c = a \star (b \star c)$  for all  $a$ ,  $b$ , and  $c$  in  $G$ .
- (b) Identity element: there must be an element  $e \in G$  such that  $e \star a = a \star e = a$  for all  $a \in G$ .
- (c) Inverses: for each  $a \in G$ , there must be an element  $a'$  such that  $a \star a' = a' \star a = e$ .

2. Let  $g$  be an element of a group  $G$ . Define the *order* of  $g$ .

Solution: The order of  $g$  is the least positive integer  $n$  such that  $g^n = e$ , where  $e$  is the identity element in  $G$ . If there is no positive integer  $n$  such that  $g^n = e$ , then the order of  $g$  is infinite.