Math 311 Solution
Quiz 2 19 September 2002

1. Here is a Cayley table for D_5, the group of symmetries of a regular pentagon. (The element R_k denotes counterclockwise rotation through k degrees; the element F_i is a reflection in the angle bisector of vertex i, where the vertices are numbered in order, going counterclockwise.)

	R_0	R_{72}	R_{144}	R_{216}	R_{288}	F_1	F_2	F_3	F_4	F_5
R_0	R_0	R_{72}	R_{144}	R_{216}	R_{288}	F_1	F_2	F_3	F_4	F_5
R_{72}	R_{72}	R_{144}	R_{216}	R_{288}	R_0	F_4	F_5	F_1	F_2	F_3
R_{144}	R_{144}	R_{216}	R_{288}	R_0	R_{72}	F_2	F_3	F_4	F_5	F_1
R_{216}	R_{216}	R_{288}	R_0	R_{72}	R_{144}	F_5	F_1	F_2	F_3	F_4
R_{288}	R_{288}	R_0	R_{72}	R_{144}	R_{216}	F_3	F_4	F_5	F_1	F_2
F_1	F_1	F_3	F_5	F_2	F_4	R_0	R_{216}	R_{72}	R_{288}	R_{144}
F_2	F_2	F_4	F_1	F_3	F_5	R_{144}	R_0	R_{216}	R_{72}	R_{288}
F_3	F_3	F_5	F_2	F_4	F_1	R_{288}	R_{144}	R_0	R_{216}	R_{72}
F_4	F_4	F_1	F_3	F_5	F_2	R_{72}	R_{288}	R_{144}	R_0	R_{216}
F_5	F_5	F_2	F_4	F_1	F_3	R_{216}	R_{72}	R_{288}	R_{144}	R_0

List the elements of $\langle R_{72}\rangle$ and the elements of $\langle F_3\rangle$.

Solution: From the table, we find that $R_{72}^2 = R_{144}$, $R_{72}^3 = R_{216}$, $R_{72}^4 = R_{288}$, and $R_{72}^5 = R_0$, which is the identity element in this group. Thus we have

$$\langle R_{72}\rangle \quad = \quad \{R_0, R_{72}, R_{144}, R_{216}, R_{288}\}.$$

Again from the table, we find that $F_3^2 = R_0$, which is the identity, so F_3 has order 2, and

$$\langle F_3\rangle \quad = \quad \{R_0, F_3\}.$$

2. Let G be a group. Define the *center* of G.

Solution: The center of G, denoted $Z(G)$, is the set of all elements x in G such that $xa = ax$ for every element $a \in G$.