

1. State the division algorithm.

Solution: Let  $a$  and  $b$  be integers with  $b > 0$ . Then there exist unique integers  $q$  and  $r$  such that

$$a = bq + r$$

with  $0 \leq r < b$ .

2. Suppose  $G$  is a cyclic group of order 33 and  $a$  is a generator of  $G$ . What does the Fundamental Theorem of Cyclic Groups say about the subgroups of  $G$ ?

Solution: The subgroups of  $G$  are all cyclic. For each divisor  $k$  of 33, there is exactly one subgroup of order  $k$ , and it is generated by  $a^{33/k}$ . In this case, the subgroups of  $G$  are

$\langle a \rangle$ , of order 33,  
 $\langle a^3 \rangle$ , of order 11,  
 $\langle a^{11} \rangle$ , of order 3, and  
 $\langle e \rangle$ , of order 1.