Math 311 | Solution
Quiz 7 | 31 October 2002

1. Let G be a group and H a subgroup of G. Let $a \in G$. Define the *left coset of H in G containing a*.

   Solution: The left coset of H in G containing a is the set

$$aH \quad = \quad \{ah : h \in H\}.$$

2. Consider the group $\mathbb{Z}_{12}$, and let $H = \{0, 3, 6, 9\}$. Then H is a subgroup of $\mathbb{Z}_{12}$. List all the (distinct) left cosets of H in $\mathbb{Z}_{12}$, and write down the elements of each one.

   Solution: There are three distinct cosets:

$$\begin{aligned} H &= \{0, 3, 6, 9\} \\ 1 + H &= \{1, 4, 7, 10\} \\ 2 + H &= \{2, 6, 8, 11\}. \end{aligned}$$

3. State Lagrange's Theorem.

   Solution: If G is a finite group and H is a subgroup of G, then $|H|$ divides $|G|$, and the number of left (or right) cosets of H in G is equal to $|G|/|H|$.