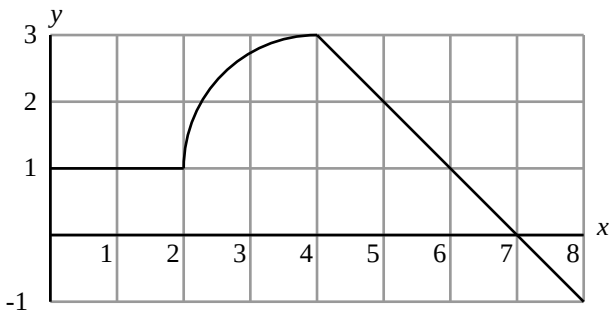
Fundamental Theorem of Calculus problems – Solutions

1. The diagram shows a graph of the function f on the interval $[0, 8]$. Let

$$g(x) = \int_0^x f(t)\,dt.$$



(a) Find the following values:

i. $g(2)$
ii. $g(6)$
iii. $g'(1)$
iv. $g'(4)$

Assume the curved part of the graph is a quarter circle.

Solution: Values of g are given by areas under the curve; values of g' can be read right off the graph.

i. $g(2) = 2$
ii. $g(6) = 8 + \pi$
iii. $g'(1) = 1$
iv. $g'(4) = 3$.

(b) Find the maximum value of g on $[0, 8]$.

Solution: The maximum occurs at $x = 7$, and the maximum value is

$$\begin{aligned} g(7) &= g(6) + \frac{1}{2} \\ &= \frac{17}{2} + \pi. \end{aligned}$$

2. Let $F(x) = \int_1^{x^4} \cos(t)\,dt$. Find $F'(x)$.

   Solution: Let $G(u) = \int_1^{u} \cos(t)\,dt$. Then

$$G'(u) = \cos(u)$$

   and $F(x) = G(x^4)$, so we get

$$\begin{aligned} F'(x) &= G'(x^4) \cdot 4x^3 \\ &= 4x^3 \cos(x^4). \end{aligned}$$