1. The diagram shows a graph of the function $f$ on the interval $[0, 8]$. Let

$$g(x) = \int_0^x f(t) \, dt.$$ 

(a) Find the following values:

i. $g(2)$

ii. $g(6)$

iii. $g'(1)$

iv. $g'(4)$

Assume the curved part of the graph is a quarter circle.

Solution: Values of $g$ are given by areas under the curve; values of $g'$ can be read right off the graph.

i. $g(2) = 2$

ii. $g(6) = 8 + \pi$

iii. $g'(1) = 1$

iv. $g'(4) = 3$.

(b) Find the maximum value of $g$ on $[0, 8]$.

Solution: The maximum occurs at $x = 7$, and the maximum value is

$$g(7) = g(6) + \frac{1}{2}$$

$$= \frac{17}{2} + \pi.$$
2. Let $F(x) = \int_1^{x^4} \cos(t) \, dt$. Find $F'(x)$.

Solution: Let $G(u) = \int_1^{u} \cos(t) \, dt$. Then

$$G'(u) = \cos(u)$$

and $F(x) = G(x^4)$, so we get

$$F'(x) = G'(x^4) \cdot 4x^3$$

$$= 4x^3 \cos(x^4).$$