Newton's approximation problems – Solutions

1. The equation $2x^3 - x - 4 = 0$ has a solution near 1.5. Using Newton's method with $x_0 = 1.5$, find x_1 and x_2. Approximate the solution to six decimal places.

   Solution: We set $f(x) = 2x^3 - x - 4$, so that $f'(x) = 6x^2 - 1$. The iteration rule for Newton's approximation is

$$\begin{aligned} x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \\ &= x_n - \frac{2x^3 - x - 4}{6x^2 - 1} \end{aligned}$$

   Using a calculator and starting with $x_0 = 1.5$, we get $x_1 = 1.4$ and $x_2 \approx 1.39182156$. The numbers for x_4 and x_5 are identical on the calculator screen, so we assume that all the digits displayed are correct. We get

$$x_4 \approx 1.391769$$

2. Use Newton's approximation with an initial guess of $x_0 = 5$ to approximate $\sqrt[3]{120}$. List the values of x_1, x_2, x_3, and x_4 to six decimal places.

   Solution: The number $\sqrt[3]{120}$ is a solution to the equation $x^3 - 120 = 0$. We set $f(x) = x^3 - 120$, and find $f'(x) = 3x^2$. The iteration rule for Newton's approximation in this case is

$$x_{n+1} = x_n - \frac{x_n^3 - 120}{3x_n^2}$$

   Using a calculator with $x_0 = 5$, we get

$$\begin{aligned} x_1 &\approx 4.933333 \\ x_2 &\approx 4.932424 \\ x_3 &\approx 4.932424 \\ x_4 &\approx 4.932424 \end{aligned}$$