Newton’s approximation problems – Solutions

1. The equation $2x^3 - x - 4 = 0$ has a solution near 1.5. Using Newton’s method with $x_0 = 1.5$, find $x_1$ and $x_2$. Approximate the solution to six decimal places.

Solution: We set $f(x) = 2x^3 - x - 4$, so that $f'(x) = 6x^2 - 1$. The iteration rule for Newton’s approximation is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{2x^3 - x - 4}{6x^2 - 1}$$

Using a calculator and starting with $x_0 = 1.5$, we get $x_1 \approx 1.4$ and $x_2 \approx 1.39182156$. The numbers for $x_4$ and $x_5$ are identical on the calculator screen, so we assume that all the digits displayed are correct. We get

$$x_4 \approx 1.391769$$

2. Use Newton’s approximation with an initial guess of $x_0 = 5$ to approximate $\sqrt[3]{120}$. List the values of $x_1$, $x_2$, $x_3$, and $x_4$ to six decimal places.

Solution: The number $\sqrt[3]{120}$ is a solution to the equation $x^3 - 120 = 0$. We set $f(x) = x^3 - 120$, and find $f'(x) = 3x^2$. The iteration rule for Newton’s approximation in this case is

$$x_{n+1} = x_n - \frac{x^3_n - 120}{3x^2_n}$$

Using a calculator with $x_0 = 5$, we get

$$x_1 \approx 4.933333$$
$$x_2 \approx 4.932424$$
$$x_3 \approx 4.932424$$
$$x_4 \approx 4.932424$$