

1. Let  $f$  be the function on the interval  $[-2, 3]$  given by

$$f(x) = \begin{cases} x + 3 & \text{if } -2 \leq x < 0 \\ 2 - \frac{x}{2} & \text{if } 0 \leq x < 2 \\ 1 - (x - 2)^2 & \text{if } 2 \leq x < 3 \end{cases}$$

- (a) Find  $\lim_{x \rightarrow 0} f(x)$ .

- (b) Find  $\lim_{x \rightarrow 2} f(x)$ .

- (c) Find all numbers  $x$  in  $[-2, 3]$  at which  $f$  is not differentiable.

2. Find the following limits algebraically. Show your work.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4}$

(b)  $\lim_{x \rightarrow -2^-} \frac{x^2 + 2x - 8}{x^2 - 4}$

(c)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 + 1})$

(d)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^4 + 8}}{(x - 3)(x - 5)}$

3. Use the definition of the derivative to find  $f'(x)$  if  $f(x) = \frac{1}{\sqrt{x}}$ .

4. Use any appropriate differentiation rules to compute  $f'(x)$ .

(a)  $f(x) = \frac{3^x}{x}$

(b)  $f(x) = \frac{\sin(2x)}{\sqrt{x}}$

(c)  $f(x) = x^3(2x^2 + 7)^{10}$

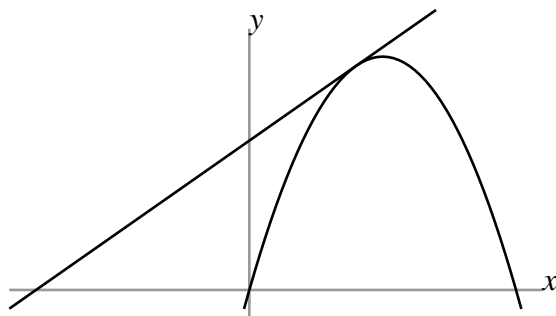
5. Suppose we invest \$1000 at a fixed, continuously-compounded interest rate  $r$  for ten years. The value of the investment at the end of the ten years depends on the interest rate  $r$ . Let  $P(r)$  denote the value of the investment (in dollars) at the end of ten years when the interest rate is  $r$  percent.

Suppose that  $P(3) = 1350$  and  $P'(3) = 135$ .

(a) What are the units of  $P'(r)$ ? What is the meaning of the statement  $P'(3) = 135$ ?

(b) What is a good estimate for  $P(3.3)$ ?

6. The line in the picture below crosses the  $x$ -axis at the point  $(-4, 0)$  and is tangent to the parabola  $y = 5x - x^2$ . Find an equation for the line. (The picture is not to scale.)



7. Suppose that  $f$  is a differentiable function, and  $G(x) = f(x^3 - 7)$ . Given that  $G(2) = 10$  and  $G'(2) = -3$ , find  $f(1)$  and  $f'(1)$ .