Math 101(02) Solution
Quiz 4 10 October 2003

1. Let $f(x) = \sqrt{3x+4}$. Use the *definition of the derivative* to find $f'(7)$.

   Solution: We have

$$\begin{aligned} f'(7) &= \lim_{h\to 0} \frac{\sqrt{3(7+h)+4}-5}{h} \\ &= \lim_{h\to 0} \frac{3(7+h)+4-25}{h(\sqrt{3(7+h)+4}+5)} \\ &= \lim_{h\to 0} \frac{3h}{h(\sqrt{25+3h}+5)} \\ &= \lim_{h\to 0} \frac{3}{\sqrt{25+3h}+5} \\ &= \frac{3}{10}. \end{aligned}$$

2. If the tangent line to $y = f(x)$ at the point $(2,6)$ passes through the point $(4,5)$, find $f'(2)$.

   Solution: The number $f'(2)$ is the slope of the tangent line. We know two points on the tangent line, so we can determine its slope as

$$\begin{aligned} f'(2) &= \frac{\text{rise}}{\text{run}} \\ &= -\frac{1}{2}. \end{aligned}$$

3. The barometric pressure $p$ (measured in inches of mercury) depends on altitude $h$ (measured in feet above sea level). Suppose $p = f(h)$.

   (a) What are the units of $f'(h)$?

   (b) If $f(2000) = 29.00$ and $f'(2000) = -0.0012$, what can you say about the barometric pressure at an altitude of 2100 feet?

   Solution:

   (a) The units are inches of mercury per foot.

   (b) The barometric pressure at 2000 feet is 29.00 in Hg, and the pressure drops about $-0.0012$ in Hg for every foot of altitude, at least when you're not far from 2000 feet. So we guess that the pressure at 2100 feet is about $100 \times (0.0012) = 0.120$ in Hg less than the pressure at 2000 feet. The pressure at 2100 feet is approximately 28.88 inches of mercury.