Math 101(02) Solution
Quiz 5 17 October 2003

1. Find the values of x at which the tangent line to the curve $y = 3x^2 - x^3 + 2$ is horizontal.

   Solution: We find that $y' = 6x - 3x^2$. The tangent line is horizontal when $y' = 0$, that is, when $6x - 3x^2 = 0$. We get

$$\begin{aligned} 0 &= 6x - 3x^2 \\ &= 3x(2 - x) \end{aligned}$$

   so the values we want are $x = 0$ and $x = 2$.

2. Let $f(x) = x^2 e^x$. Find an equation for the line tangent to the curve $y = f(x)$ at the point $(1, e)$.

   Solution: We have

$$\begin{aligned} f'(x) &= x^2 e^x + 2xe^x \\ f'(1) &= e + 2e \\ &= 3e. \end{aligned}$$

   The slope of the tangent line is $3e$. It contains the point $(1, e)$, so we may use the point-slope form to write

$$y - e = 3e(x - 1)$$

3. Find $\dfrac{d}{dt}\left(\dfrac{t^3 - t}{2t^2 + 5}\right)$. Do not simplify.

   Solution: We apply the quotient rule, getting

$$\frac{d}{dt}\left(\frac{t^3 - t}{2t^2 + 5}\right) = \frac{(2t^2 + 5)(3t^2 - 1) - (t^3 - t)(4t)}{(2t^2 + 5)^2}.$$