Math 101(02) Solution
Quiz 6 31 October 2003

1. Find the slope of the line tangent to the curve $x^3 + 3y^2 = xe^y$ at the point $(1, 0)$.

   Solution: We take derivatives with respect to x to get

$$\begin{aligned} 3x^2 + 6y\frac{dy}{dx} &= e^y + xe^y\frac{dy}{dx} \\ (6y - xe^y)\frac{dy}{dx} &= e^y - 3x^2 \\ \frac{dy}{dx} &= \frac{e^y - 3x^2}{6y - xe^y}. \end{aligned}$$

   At the point $(1, 0)$, we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{1-3}{-1} \\ &= 2. \end{aligned}$$

2. Fill in the blank:

$$\frac{d^2}{dx^2}\left(\frac{1}{1+x^2}\right) = \frac{\boxed{}}{(1+x^2)^3}.$$

   Solution: We have $\frac{d}{dx}(1+x^2)^{-1} = -(1+x^2)^{-2} \cdot 2x$, and

$$\begin{aligned} \frac{d}{dx}(-2x(1+x^2)^{-2}) &= -2x(-2(1+x^2)^{-3}(2x)) + (-2)(1+x^2)^{-2} \\ &= 8x^2(1+x^2)^{-3} - 2(1+x^2)^{-2} \\ &= \frac{8x^2 - 2(1+x^2)}{(1+x^2)^3} \\ &= \frac{\boxed{6x^2 - 2}}{(1+x^2)^3}. \end{aligned}$$

3. Use logarithmic differentiation to find y' if $y = (2x^2+1)^5\sqrt[3]{4-x}$.

   Solution: We take logarithms to get

$$\ln y = 5\ln(2x^2+1) + \frac{1}{3}\ln(4-x).$$

   We differentiate to get

$$\frac{1}{y}y' = \frac{20x}{2x^2+1} - \frac{1}{3(4-x)}$$

   so that

$$\begin{aligned} y' &= y\left[\frac{20x}{2x^2+1} - \frac{1}{3(4-x)}\right] \\ &= (2x^2+1)^5\sqrt[3]{4-x}\left[\frac{20x}{2x^2+1} - \frac{1}{3(4-x)}\right]. \end{aligned}$$