Math 101(02) | Solution
Quiz 7 | 7 November 2003

1. Complete the definition: A function f with domain D has an *absolute maximum* at c if ... (Solution:) ... $f(c) \geq f(x)$ for all x in D.

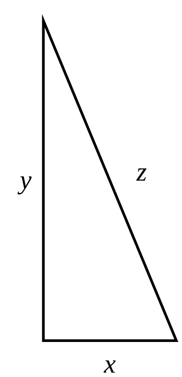
2. Find the linearization $L(x)$ for the function $f(x) = \sqrt{x}$ at $a = 121$ and use it to approximate $\sqrt{120}$.

   Solution: First we observe that $f(121) = \sqrt{121} = 11$. We have $f'(x) = \dfrac{1}{2\sqrt{x}}$ so $f'(121) = \dfrac{1}{22}$. The linearization we want is given by $L(x) = 11 + \dfrac{1}{22}(x - 121)$.

   We approximate $\sqrt{120}$ as $L(120) = 11 - \dfrac{1}{22} = \dfrac{241}{22}$.

3. Cyril the ant crawls toward the origin along the positive y-axis at 2 units per second, while Evelyn the snail slides away from the origin along the positive x-axis at $\frac{1}{2}$ unit per second. How fast is the distance between Cyril and Evelyn changing when Cyril is at the point $(0, 12)$ and Evelyn is at the point $(5, 0)$? Is the distance increasing or decreasing?

   Solution: Let y denote Cyril's y-coordinate and x denote Evelyn's x coordinate. Let z denote the distance between Cyril and Evelyn. We are given that $\dfrac{dy}{dt} = -2$ and $\dfrac{dx}{dt} = \dfrac{1}{2}$ and asked to find $\dfrac{dz}{dt}$. We have



$$z^2 = x^2 + y^2$$

   so that

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}.$$

   When $y = 12$ and $x = 5$, we get $z = 13$, so our equation becomes

$$\begin{aligned} 26\frac{dz}{dt} &= 10\left(\frac{1}{2}\right) - 24(2) \\ &= -43 \end{aligned}$$

   so that $\dfrac{dz}{dt} = -\dfrac{43}{26}$. The distance between Cyril and Evelyn is decreasing at $\dfrac{43}{26}$ units per second.