Math 101(02) Solution
Quiz 8 14 November 2003

1. Find $\lim_{x\to 0} \frac{\ln(1-2x)}{x}$.

   Solution: The form of the limit is $0/0$, so we may apply l'Hospital's rule to get

$$\begin{aligned}\lim_{x\to 0} \frac{\ln(1-2x)}{x} &= \lim_{x\to 0} \frac{\frac{-2}{1-2x}}{1} \\ &= -2\end{aligned}$$

2. Find $\lim_{x\to\infty} xe^{-x}$.

   Solution: The form of the limit is $\infty \cdot 0$. We rewrite it as $\lim_{x\to\infty} \frac{x}{e^x}$, which has form ∞/∞, and apply l'Hospital's rule to get

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x}$$

   The latter limit has form $1/\infty$, so it is 0.

3. Find the absolute maximum and minimum values of the function $f(x) = 15x - x^3 + 5$ on the interval $[0, 3]$. Give your answers in exact form.

   We have $f'(x) = 15 - 3x^2 = 3(5 - x^2)$, so there are critical number at $x = \pm\sqrt{5}$. Only the critical number $\sqrt{5}$ lies within the given interval. We have

$$f(0) = 5; \quad f(\sqrt{5}) = 15\sqrt{5} - 5\sqrt{5} + 5 = 10\sqrt{5} + 5; \quad f(3) = 45 - 27 + 5 = 23$$

   The largest of these values is $10\sqrt{5} + 5$, so that is the absolute maximum value. The smallest is 5, so that is the absolute minimum value.