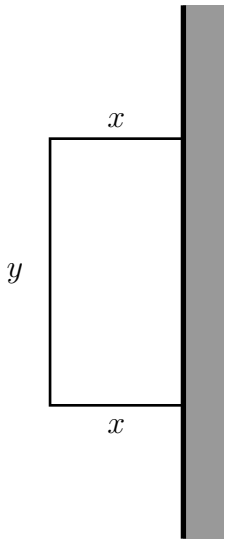
Math 101(02) Solution
Quiz 9 5 December 2003

1. A farmer wants to build a rectangular pen for some lemmings. One side of the pen will be formed by the wall of a large barn; for the other three sides, the farmer will put up fencing. What is the area of the largest rectangular pen the farmer can build if she has 36 feet of fencing available?

   Solution: Let x and y be the width and length of the pen, as in the diagram at right. We want to maximize $A = xy$, and we know that $2x + y = 36$, so that $y = 36 - 2x$. Writing a formula for A in terms of x alone, we get

$$\begin{aligned} A(x) &= x(36 - 2x) \\ &= 36x - 2x^2 \end{aligned}$$

   We want to find the maximum of A for $0 < x < 18$. To do this, we compute

$$A'(x) = 36 - 4x$$

   and find that A has a critical number at $x = 9$. Since $A'(x) > 0$ for $0 < x < 9$ and $A'(x) < 0$ for $9 < x < 18$, we conclude that A has an absolute maximum at $x = 9$. The maximum area is

$$A(9) = 9(36 - 18) = 162 \text{ square feet}$$

2. The function $f(x) = x^4 - x^3 - 2$ has a root near $x = 1.5$. Using Newton's method with an initial guess of $x_0 = 1.5$, find x_1 and x_2. Give your answers to six decimal places.

   Solution: We set up the iteration rule for Newton as

$$x_{n+1} = x_n - \frac{x_n^4 - x_n^3 - 2}{4x_n^3 - 3x_n^2}$$

   Taking $x_0 = 1.5$, we get

$$\begin{aligned} x_1 &\approx 1.546296 \\ x_2 &\approx 1.543698 \end{aligned}$$