Math 101 Writing Assignment #3 Due 18 November 2003

Assignment: Write, in complete sentences, a solution to Section 3.10, Exercise 17:

> At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

Solution: Let x be the distance that ship A has sailed from its position at noon, and let y be the distance that ship B has sailed from its position at noon. Let z be the distance between the two ships. We'll measure all distances in kilometers.

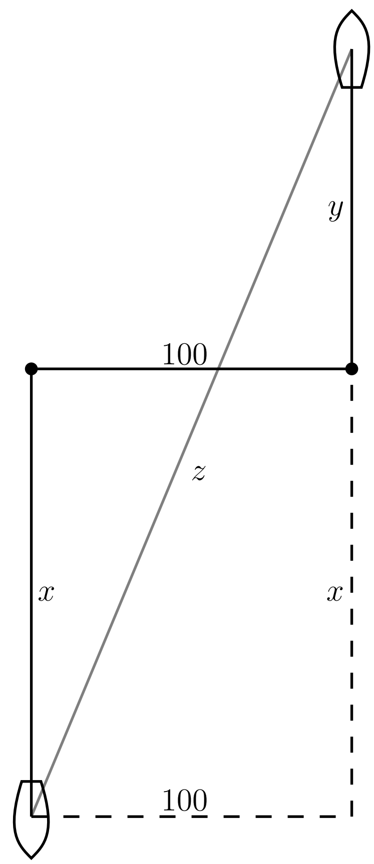
We are given that $\frac{dx}{dt} = 35$ km/h and $\frac{dy}{dt} =$ 25 km/h. We are asked to find $\frac{dz}{dt}$ at 4:00 PM.

In the diagram, we note that there is a right triangle with hypotenuse z and legs 100 and $(x + y)$. Thus we have the equation

$$z^2 = 100^2 + (x+y)^2$$

relating x, y, and z. We differentiate both sides of this equation with respect to t to get

$$2z\frac{dz}{dt} = 2(x+y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) \qquad (1)$$



We know the values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$. We need to find the values of x, y, and z at 4:00 PM. Between noon and 4:00 PM, ship A sailed south at a steady

speed of 35 km/h, so at 4:00 PM, ship A is

$$4 \times 35 = 140 \text{ km}$$

south of its position at noon. That is, $x = 140$ at 4:00 PM. By a similar calculation, ship B is

$$4 \times 25 = 100 \text{ km}$$

north of its noontime position at 4:00 PM, so we have $y = 100$ at 4:00 PM. To find the value of z at 4:00 PM, we substitute $x = 140$ and $y = 100$ into the equation

$$z^2 = 100^2 + (x+y)^2$$

to get

$$\begin{aligned} z^2 &= 100^2 + 240^2 \\ &= (5 \times 20)^2 + (12 \times 20)^2 \\ &= (5^2 + 12^2) \times 20^2 \\ &= 13^2 \times 20^2 \end{aligned}$$

so that $z = 13 \times 20 = 260$ at 4:00 PM.

We now substitute $x = 140$, $y = 100$, $z = 260$, and the given values for $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ into equation (1) to get

$$\begin{aligned} 2(260)\frac{dz}{dt} &= 2(140+100)(35+25) \\ \frac{dz}{dt} &= \frac{2(240)(60)}{2(260)} \\ &= \frac{720}{13} \end{aligned}$$

At 4:00 PM, the distance between ship A and ship B is increasing at the rate of $\dfrac{720}{13}\dfrac{\text{km}}{\text{h}}$, or about 55.38 km/h.