Math 203 Practice Exam I October 2003

1. Let P be the point $(0, 2, 1)$, and Q be the point $(-1, 3, 5)$.

   (a) Write a vector parametrization of the line segment $\overline{PQ}$.

   (b) Find the area and the perimeter of triangle whose vertices are P, Q, and the origin.

2. Let p_1 be the plane $2x + 4y - z = 5$ and p_2 be the plane $x - 3y - 2z = 2$.

   (a) Find a vector parametrization for the line of intersection of p_1 and p_2.

   (b) Find the cosine of the angle of intersection of the planes p_1 and p_2.

3. Let ℓ be the line with parametric equations

$$\begin{aligned} x &= 2t \\ y &= -2t \\ z &= -t \end{aligned}$$

and let $\vec{v} = \langle 2, -5, 6 \rangle$. Find vectors $\vec{a}$ and $\vec{b}$ so that $\vec{a}$ is parallel to ℓ, $\vec{b}$ is perpendicular to ℓ, and $\vec{a} + \vec{b} = \vec{v}$.

4. Identify and sketch the quadric surface $x^2 + y^2 + 2y - z^2 - 4z = 0$.

5. (a) Find the cylindrical and rectangular coordinates for the point whose spherical coordinates are $\rho = 3$, $\theta = \dfrac{\pi}{4}$, $\varphi = \dfrac{\pi}{6}$.

(b) Sketch the region described by $0 \leq \theta \leq \dfrac{\pi}{2}$, $0 \leq r \leq 1$, $r^2 \leq z \leq 1$.

6. My motorcycle gets about 50 miles to the gallon. I use the trip odometer to keep track of how far I've ridden between refuelings. When the trip odometer reads about 100 miles, I stop for fuel, and the tank usually takes about 2 gallons.

   Of course, the odometer doesn't always read exactly 100 miles at each fuel stop (there might not be a gas station right at that place in the road), and the tank doesn't always take exactly 2 gallons.

   Let $f(m, g)$ denote the motorcycle's fuel economy (in miles per gallon) as a function of m, the number of miles driven, and g the number of gallons of fuel used. Find a linear function $L(m, g)$ that best approximates f when m is near 100 and g is near 2.

7. Here is a contour plot of a function $f(x, y)$. Use the contour plot to construct a reasonable linear approximation to f at the point $(-1, -1)$.