Math 203 | Practice Exam II | 10 November 2003

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1. Let $f(x, y, z) = 3x^2z - 2y^2$. Let P be the point $(4, 1, 2)$. Find the directional derivative of f at P in the direction from P toward the x-axis.

2. Sand runs into the lower globe of an hourglass at 2 cm^3/min and forms a pile in the shape of a right circular cone. After about two minutes and twenty seconds, the height of the pile reaches 2 cm and the radius (of the base) reaches 1.5 cm. Suppose that the height of the pile is increasing at 0.2 cm/sec at that moment. How fast is the radius increasing at the same moment?

3. Find and classify all the critical points of the function $f(x, y) = x^2y^2 - 4x^2 - y^2 + 12y$.

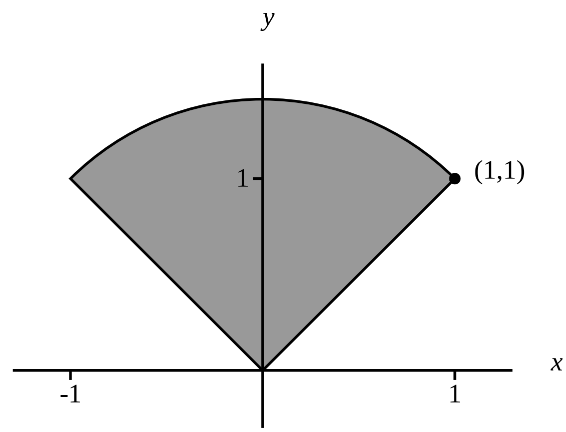
4. Let R be the region in the xy-plane with $x \geq 0$, $y \geq 0$, and $x + y \leq 4$. Find the minimum and maximum values of the function $f(x, y) = x^2 - 2x + y^2 - 4y$ on the region R.

5. Use the method of Lagrange multipliers to find the points on the ellipse

$$(x-8)^2 + 9y^2 = 117$$

that are closest to the origin.

6. Let R be the region in the xy-plane shown at right. Assume the curve is part of a circle centered at the origin. Set up, but do not evaluate, the integral $\iint_R y + 1\, dA$ in two ways:



(a) In rectangular coordinates.

(b) In polar coordinates.

7. Let $E = \{(x, y, z) \,:\, x \geq 0,\ y \geq 0,\ z \geq 0,\ x^2 + y^2 \leq 9$ and $x^2 + y^2 + z^2 \leq 25\}$. Set up (but do not evaluate) $\displaystyle\iiint\limits_E x\,dV$ in three ways:

(a) In rectangular coordinates.

(b) In cylindrical coordinates.

(c) In spherical coordinates.