Math 203 **November 2003**

Practice Problems after Hour Exam II

1. Compute

$$\int_0^1 \int_{y^2}^1 e^{x^{\frac{3}{2}}}\, dx\, dy.$$

2. Let C be the helix centered on the z-axis, circling counterclockwise, making two full rotations while rising from $(4, 0, 0)$ to $(4, 0, 8)$.

   (a) Parametrize C.

   (b) Let $\mathbf{F}(x, y, z) = -y\vec{\imath} + x\vec{\jmath} - z\vec{k}$. Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.

   (c) Let $\mathbf{G} = 2xy^2z^2\vec{\imath} + 2yx^2z^2\vec{\jmath} + (2zx^2y^2 + 1)\vec{k}$. Find $\int_C \mathbf{G} \cdot d\mathbf{r}$.

3. Let S be the part of the paraboloid $z = 9 - x^2 - y^2$ between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

   (a) Write a parametrization for S. Set it up so that the domain is a rectangle in the uv-plane.

   (b) Set up, but do not evaluate, an integral for the surface area of S.

4. Let C_1 be the half-circle $x^2 + y^2 = 4$, $y \geq 0$, $z = 0$, oriented counterclockwise from above. Let C_2 be the line segment from $(-2, 0, 0)$ to $(2, 0, 0)$. Let C be the union of these two curves, so that C is a closed curve.

   Let $\mathbf{F} = (2 - 3y)\vec{\imath} + 2x\vec{\jmath} + xy\vec{k}$.

   (a) Compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

   (b) Compute curl $\mathbf{F}$.

   (c) Use Stokes's theorem to verify your answer to part 4a.

5. Let W be the solid region $x^2 + y^2 \leq 4$ and $0 \leq z \leq 5$. Let

$$\mathbf{F}(x, y, z) = (2x + y\cos(z))\vec{\imath} + (3y + x\sin(z))\vec{\jmath} + z(5 - z)\vec{k}.$$

(a) Explain why the flux of $\mathbf{F}$ through the top and bottom faces of W is zero.

(b) Use the divergence theorem to compute the total flux of $\mathbf{F}$ through the boundary of W.

(c) Determine the flux of $\mathbf{F}$ outward through the lateral surface of the boundary of W (that is, the part of the surface that isn't the top or the bottom).

6. Let S_1 be hemisphere $x^2+y^2+z^2=25$, $z \geq 0$, oriented upward (that is, in the positive z direction). Let S_2 be the disk $x^2+y^2 \leq 25$, $z=0$, oriented downward (that is, in the negative z direction).

   Let S be the union of S_1 and S_2, and let W be the region bounded by S.

   Let

$$\mathbf{F} \quad = \quad x^2\vec{\imath} + (2y + z\cos(x))\vec{\jmath} + (y+1)\vec{k}.$$

(a) Calculate $\displaystyle\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$.

(b) Use the divergence theorem to compute $\displaystyle\iint_{S} \mathbf{F} \cdot d\mathbf{S}$.

(c) Find $\displaystyle\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$.