Math 203 Solution
Quiz 1 12 September 2002

1. Find the center and radius of the sphere with equation $x^2 + 4x + y^2 + z^2 - 2z = 5$.

   Solution: Completing the squares, we get

$$\begin{aligned} x^2 + 4x + 4 + y^2 + z^2 - 2z + 1 &= 10 \\ (x+2)^2 + y^2 + (z-1)^2 &= 10 \end{aligned}$$

   showing that the center of this sphere is $(-2, 0, 1)$ and its radius is $\sqrt{10}$.

2. Let $\vec{a} = 3\vec{\imath} + 2\vec{\jmath} - 5\vec{k}$ and $\vec{b} = -\vec{\imath} + 6\vec{\jmath} + \vec{k}$. Find the cosine of the angle between $\vec{a}$ and $\vec{b}$. Leave your answer in exact form.

   Solution: Let θ denote the angle between $\vec{a}$ and $\vec{b}$. Then

$$\begin{aligned} \cos\theta &= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} \\ &= \frac{-3 + 12 - 5}{\sqrt{38}\sqrt{38}} \\ &= \frac{4}{38} \\ &= \frac{2}{19}. \end{aligned}$$

3. Find a vector perpendicular to the plane that contains the points $P(1, 1, 5)$, $Q(4, -2, 0)$, and $R(6, 5, -1)$.

   Solution: We have

$$\begin{aligned} \overrightarrow{PQ} &= \langle 3, -3, -5 \rangle \\ \overrightarrow{PR} &= \langle 5, 4, -6 \rangle \end{aligned}$$

   so that

$$\begin{aligned} \overrightarrow{PQ} \times \overrightarrow{PR} &= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 3 & -3 & -5 \\ 5 & 4 & -6 \end{vmatrix} \\ &= 38\vec{\imath} - 7\vec{\jmath} + 27\vec{k}. \end{aligned}$$