Math 203 — Solution

Quiz 3 — 26 September 2002

1. Let $\vec{r}(t) = \langle t-2, 2-t^2, t\cos(\pi t)\rangle$. Write a set of parametric equations for the line tangent to the curve $\vec{r}(t)$ at the point where the curve crosses the yz-plane.

   Solution: The curve crosses the yz-plane at the point where its x-coordinate is 0; that is, when $t = 2$. We have

   $$\vec{r}(2) = (0, -2, 2)$$

   and

   $$\vec{r}'(t) = \langle 1, -2t, \cos(\pi t) - \pi\sin(\pi t)\rangle$$

   so that

   $$\vec{(r)}'(2) = \langle 1, -4, 1\rangle.$$

   The parametric equations are

   $$\begin{aligned} x &= 0 + t \\ y &= -2 - 4t \\ z &= 2 + t. \end{aligned}$$

2. Let $\vec{r}(t) = 2e^t\vec{\imath} - t\vec{\jmath} + e^{2t}\vec{k}$. Find the arc length of the space curve traced out by $\vec{r}$ for $0 \le t \le 2$. Leave your answer in the form of a definite integral.

   Solution: We have

   $$\begin{aligned} \vec{r}'(t) &= 2e^t\vec{\imath} - \vec{\jmath} + 2e^{2t}\vec{k} \\ |\vec{r}'(t)| &= \sqrt{4e^{2t} + 4e^{4t} + 1}. \end{aligned}$$

   The length L is given by

   $$L = \int_0^2 \sqrt{4e^{4t} + 4e^{2t} + 1}\,dt.$$

   Extra credit (2 points): Evaluate the integral.

   Since $4e^{4t} + 4e^{2t} + 1 = (2e^{2t} + 1)^2$, we have

   $$\begin{aligned} L &= \int_0^2 2e^{2t} + 1\,dt \\ &= \left[e^{2t} + t\right]_0^2 \\ &= e^4 + 1. \end{aligned}$$

3. Fill in the blanks: The surface with equation $(x-2)^2 - (y+1)^2 - (z-3)^2 + 4 = 0$ is a hyperboloid of <u>one sheet</u> with center point <u>$(2, -1, 3)$</u> and axis <u>parallel to the x-axis through the point $(0, -1, 3)$.</u>