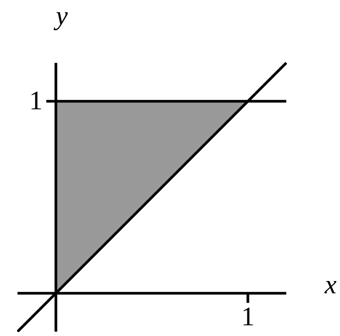
Math 203 | Solution
Quiz 7 | 31 October 2003

1. Let R be the region in the xy-plane bounded by the lines $x = 0$, $y = 1$ and $x = y$. Find $\iint_R e^{-y^2}\, dA$

   Solution: We have

$$\begin{aligned}
\iint_R e^{-y^2}\, dA &= \int_0^1 \int_0^y e^{-y^2}\, dx\, dy \\
&= \int_0^1 y e^{-y^2}\, dy \\
&= \left[-\frac{1}{2}e^{-y^2}\right]_0^1 \\
&= -\frac{1}{2}e^{-1} + \frac{1}{2} \\
&= \frac{1}{2}(1 - e^{-1}).
\end{aligned}$$



2. Find the volume of the region outside the cylinder $x^2 + y^2 = 1$, under the paraboloid $z = 4 - x^2 - y^2$, and above the xy-plane.

   Solution: The footprint is an annulus, most easily described using polar coordinates: $0 \le \theta \le 2\pi$ and $1 \le r \le 2$.

   We get

$$\begin{aligned}
V &= \int_0^{2\pi} \int_1^2 (4 - r^2) r\, dr\, d\theta \\
&= \int_0^{2\pi} \int_1^2 4r - r^3\, dr\, d\theta \\
&= \int_0^{2\pi} \left[2r^2 - \frac{r^4}{4}\right]_1^2 d\theta \\
&= \int_2^{2\pi} (8 - 4) - \left(2 - \frac{1}{4}\right) d\theta \\
&= 2\pi \left(\frac{9}{4}\right) \\
&= \frac{9\pi}{2}.
\end{aligned}$$