Math 203 Solution
Quiz 8 7 November 2003

1. Set up, but do not evaluate, an integral for the surface area of the part of the surface $z = xy$ that lies inside the cylinder $x^2 + y^2 = 9$.

   Solution: Taking $f(x, y) = xy$, we have $f_x(x, y) = y$ and $f_y(x, y) = x$, so that $\sqrt{1 + (f_x)^2 + (f_y)^2} = \sqrt{1 + y^2 + x^2}$. It's easiest to set up the integral in polar coordinates. We have

$$A = \int_0^{2\pi} \int_0^3 (\sqrt{1 + r^2})\, r\, dr\, d\theta.$$

2. Evaluate $\iiint\limits_E 4y\, dV$ where E is the region lying above the rectangle $[0, 1] \times [0, 2]$ and below the parabolic cylinder $z = 1 - x^2$.

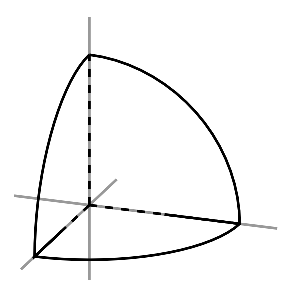
   Solution: We have

$$\begin{aligned}
\int_0^1 \int_0^2 \int_0^{1-x^2} 4y\, dz\, dy\, dx &= \int_0^1 \int_0^2 4y(1 - x^2)\, dy\, dx \\
&= \int_0^1 \left[2y^2(1 - x^2)\right]_{y=0}^2 dx \\
&= \int_0^1 8(1 - x^2)\, dx \\
&= \left[8x - \frac{8x^3}{3}\right]_{x=0}^1 \\
&= \frac{16}{3}.
\end{aligned}$$

3. Rewrite the integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} (x-y)\, dz\, dy\, dx$ in either cylindrical or spherical coordinates.

   Solution: The region of integration is the first octant of a ball of radius 2 centered at the origin. In cylindrical coordinates we have

$$\int_0^{\frac{\pi}{2}} \int_0^2 \int_0^{\sqrt{4-r^2}} r(\cos\theta - \sin\theta) r\, dz\, dr\, d\theta.$$



   In spherical coordinates, the integral is even simpler; it's

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 \rho \sin\phi(\cos\theta - \sin\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta.$$