Math 203 — Solution
Quiz 9 — 21 November 2003

1. Compute $\int_C y\,ds$ where C is the semicircle $x^2 + y^2 = 4$, $y \geq 0$.

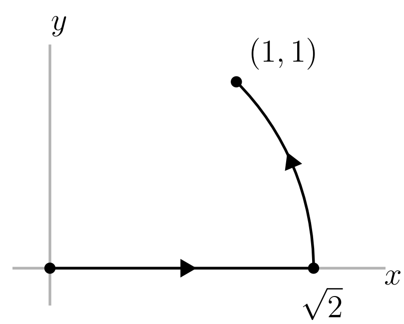
   Solution: We parametrize C as $\mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle$, $0 \leq t \leq \pi$. We have

   $$\mathbf{r}'(t) = \langle -2\sin t, 2\cos t\rangle$$

   so that $|\mathbf{r}'(t)| = \sqrt{4\sin^2 t + 4\cos^2 t} = 2$. The integral is computed as

   $$\begin{aligned} \int_C y\,ds &= \int_0^\pi 2\sin t \cdot 2\,dt \\ &= 4\int_0^\pi \sin t\,dt \\ &= -4\cos t\Big]_0^\pi \\ &= 8 \end{aligned}$$

2. Let $\mathbf{F}(x,y) = 2xe^y\vec{\imath} + (x^2e^y + 1)\vec{\jmath}$. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$ along the path from the origin to the point $(1,1)$ shown at right.



   Solution: We observe that $\mathbf{F} = \nabla f$, where $f(x,y) = x^2e^y + y$. Thus

   $$\begin{aligned} \int_C \mathbf{F}\cdot d\mathbf{r} &= f(1,1) - f(0,0) \\ &= (e+1) - 0 \\ &= e + 1. \end{aligned}$$

3. Let $\mathbf{F}(x,y,z) = \langle y^2 + z^2, x + z, yz\rangle$. Find curl $\mathbf{F}$.

   Solution: We have

   $$\begin{aligned} \text{curl } \mathbf{F} &= \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2 + z^2 & x + z & yz \end{vmatrix} \\ &= \langle z - 1, 2z, 1 - 2y\rangle \end{aligned}$$