

**Modular addition and subtraction**

**Overview.** These are some exercises in modular addition and subtraction.

**Things to do.**

1. You should be able to do these problems in your head.

$$5 + 6 \bmod 7 = \boxed{\phantom{00}} \qquad 8 + 7 \bmod 9 = \boxed{\phantom{00}} \qquad 3 - 8 \bmod 15 = \boxed{\phantom{00}}$$

2. When using a calculator to reduce a number  $x$  modulo some integer  $m$ , it's always best to take the integer part of  $x/m$ , multiply that by  $m$ , and then subtract the result from  $x$ . To reduce 365 modulo 29, for example, I'd enter

$$365 - 29 * \text{int}(365/29)$$

(the `int` function on a TI-83 is on the [MATH] [NUM] menu).

Use a calculator to do these problems.

$$1248 \bmod 5 = \boxed{\phantom{00}} \qquad 753 \bmod 27 = \boxed{\phantom{00}} \qquad -260 \bmod 39 = \boxed{\phantom{00}}$$

The alternative method for reducing mod  $m$  on a calculator is to take the fractional part of  $x/m$  and then multiply it by  $m$ . This looks okay for small numbers, but can lead to inaccurate results because of rounding problems. Calculate  $1,234,567,890 \bmod 7$  in two ways:

$$\begin{aligned} 7 * \text{fPart}(1234567890/7) &= \boxed{\phantom{00000000}} \\ 1234567890 - 7 * \text{int}(1234567890/7) &= \boxed{\phantom{00000000}} \end{aligned}$$

3. Fill in this mod-6 addition table. You can probably do all the addition in your head.

+	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

4. Use Excel to make a mod-26 addition table. Here's how.
- Start with a new worksheet. Click on the grey cell at the top right to highlight all the cells in the worksheet. Choose the menu item **Format>Columns>Width**, and type "2" to set the column width to 2 throughout the worksheet.
  - Enter a 0 into cell B1 and a 1 into cell C1. Highlight both these cells. Now pick up the small black square at the lower right corner of the highlight, and drag it over to cell AA1. When you let go of the box, you should see the numbers 0 through 25 in the top row of your worksheet.
  - Use the same technique to enter the numbers 0 through 25 in cells A2 through A28.
  - Click on the grey "1" cell at the left to highlight row 1. Click on the boldface "B" on the toolbar to change the text in row 1 into boldface. Use the same technique to change the text in column A to boldface.
  - Now enter the formula `=mod(B$1+$A2,26)` into cell B2. (The dollar signs denote absolute references. In this case, for example, the reference **B\$1** means "look in *this* column (because we happen to be in column B), and in row 1.")
  - Highlight cell B2, grab the small black square at the lower right corner, and drag it to cell AA2. This should fill in the first row of your table. Now grab the small black square again, and drag it *down* to cell AA28. This should fill in the whole table.
5. Obtain another cipher disk, and fill in each ring with numbers (0 – 25, in order) instead of letters. Try to figure out how to use your cipher disk to do addition modulo 26. Make up some addition problems, solve them with the disk, and then check your answers against the Excel table.