Math 202 **Notes on Partial Fractions** **21 February 2002**

Here's how to compute $\int \frac{P(x)}{Q(x)}\,dx$ for any pair of polynomials P and Q.

1. Start with a rational function of the form $\frac{P(x)}{Q(x)}$ with $\deg P(x) < \deg Q(x)$.

   You may need to use polynomial division to make $\deg P(x) < \deg Q(x)$.

2. Factor $Q(x)$ into linear factors and irreducible quadratic factors.

   A linear factor has the form $(ax + b)$ for some constants a and b. An irreducible quadratic factor has the form $(ax^2 + bx + c)$ for some constants a, b, and c, with $b^2 - 4ac < 0$.

3. Set $\frac{P(x)}{Q(x)}$ equal to a sum of partial fractions terms, using the factors of $Q(x)$ to determine what terms to write.

   For each linear factor $(ax + b)$, write a term of the form $\frac{A}{(ax+b)}$.

   For each repeated linear factor $(ax + b)^n$, write n terms:

   $$\frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \cdots + \frac{A_n}{(ax+b)^n}.$$

   For each irreducible quadratic factor $(ax^2 + bx + c)$, write a term $\frac{Bx+C}{(ax^2+bx+c)}$.

   For each repeated irreducible quadratic factor $(ax^2 + bx + c)^n$, write n terms:

   $$\frac{B_1x+C_1}{(ax^2+bx+c)} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots \frac{B_nx+C_n}{(ax^2+bx+c)^n}.$$

4. Multiply through to clear denominators, expand all expressions, and equate coefficients. That is, for each exponent n, set the coefficient of x^n on the left side of the equation equal to the coefficient of x^n on the right side.

5. Solve the resulting system for A, B, C, and so on.

6. Substitute the values you found for A, B, C, and so on back into the partial fractions terms you wrote in step 3, and integrate one term at a time.