Here’s how to compute \( \int \frac{P(x)}{Q(x)} \, dx \) for any pair of polynomials \( P \) and \( Q \).

1. Start with a rational function of the form \( \frac{P(x)}{Q(x)} \) with \( \deg P(x) < \deg Q(x) \).
   
   You may need to use polynomial division to make \( \deg P(x) < \deg Q(x) \).

2. Factor \( Q(x) \) into linear factors and irreducible quadratic factors.
   
   A linear factor has the form \((ax + b)\) for some constants \(a\) and \(b\). An irreducible quadratic factor has the form \((ax^2 + bx + c)\) for some constants \(a\), \(b\), and \(c\), with \(b^2 - 4ac < 0\).

3. Set \( \frac{P(x)}{Q(x)} \) equal to a sum of partial fractions terms, using the factors of \( Q(x) \) to determine what terms to write.

   For each linear factor \((ax + b)\), write a term of the form \( \frac{A}{(ax + b)} \).

   For each repeated linear factor \((ax + b)^n\), write \(n\) terms:
   
   \[
   \frac{A_1}{(ax + b)} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_n}{(ax + b)^n}.
   \]

   For each irreducible quadratic factor \((ax^2 + bx + c)\), write a term \( \frac{Bx + C}{(ax^2 + bx + c)} \).

   For each repeated irreducible quadratic factor \((ax^2 + bx + c)^n\), write \(n\) terms:
   
   \[
   \frac{B_1x + C_1}{(ax^2 + bx + c)} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}.
   \]

4. Multiply through to clear denominators, expand all expressions, and equate coefficients.
   
   That is, for each exponent \(n\), set the coefficient of \(x^n\) on the left side of the equation equal to the coefficient of \(x^n\) on the right side.

5. Solve the resulting system for \( A \), \( B \), \( C \), and so on.

6. Substitute the values you found for \( A \), \( B \), \( C \), and so on back into the partial fractions terms you wrote in step 3, and integrate one term at a time.