Math 202 Practice Exam 3 22 April 2002

Instructions: This is a self-timed, take-home exam. The time limit is 90 minutes. This means that you must stop working on the exam ninety minutes after you first look at the exam questions.

You may use one single-sided sheet of notes on this exam. You will also want to use a calculator that can compute sums of sequences. Otherwise, this is a closed-book exam.

1. (a) Let R be the region bounded by the curve $y = 4 - x^2$ and the x-axis. Set up, but do not evaluate, an integral for the volume generated when R is revolved about the line $y = 4$.

   (b) Let R be the region bounded by the parabola $y = (x-1)^2$ and the line $y = 1$. Set up, but do not evaluate, an integral for the volume generated when R is revolved about the y-axis.

2. Set up, but do not evaluate, an integral for the area of the surface generated when the part of the curve $y = (\ln x)^2$ with $1 \le x \le 4$ is revolved about the x-axis.

3. (a) Consider the sequence $\{a_n\}$ given by $a_n = 3^n 4^{1-n}$.
   Find $\lim_{n\to\infty} a_n$.

   (b) Consider the sequence $\{a_n\}$ given by $a_n = \dfrac{\sin n}{n^2}$
   Find $\lim_{n\to\infty} a_n$.

4. (a) Find the sum of the series
   $$\frac{3}{4} - \frac{6}{12} + \frac{12}{36} - \frac{24}{108} + \cdots,$$
   assuming the pattern continues.

   (b) Let $x = 3.513513513513\ldots$. Write x as a quotient of two integers.

5. Determine whether each series converges or diverges. Give reasons.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{n}{3n+5}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^n + 2}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n-1}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n(-1)^{n-1}}{2^n}$

6. Let $s = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$. (The series is convergent by the Alternating Series Test.) How many terms of the series are necessary to estimate s with an error of less than 0.001? Use a calculator to estimate s with an error of less than 0.001.