Practice Problems **6 May 2002**

These problems cover the material after Hour Exam 3. The Final Exam will be comprehensive, and will include material from Hour Exams 1, 2, and 3, as well as material covered in these problems.

1. Determine whether the given series is absolutely convergent, conditionally convergent, or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{(-2)^n}$.

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$.

   (c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{3-n^2}{3n^2+2}\right)^n$.

2. Find a power-series representation of the function $x \ln(2+x)$. Be sure to indicate the interval on which your power series is valid.

3. Determine the interval of convergence for the given power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n)!}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{3^n \sqrt{n}}$.

4. Use a power series to estimate the value of $\displaystyle\int_0^{\frac{1}{2}} \frac{x}{1+x^4}\, dx$ with an error of less than 10^{-6}.

5. (a) Find the third-order Taylor polynomial for $f(x) = \tan x$ about $\pi/4$.

   (b) Find the Taylor series for $f(x) = \sqrt{x+1}$ at 0. Write the first three terms and the general form of the n^{th} term.