Math 202(01) Solution
Quiz 2 8 February 2002

Write all answers in exact form. Draw boxes around your final answers.

1. Find $\lim_{x\to 0} \frac{\cos x - 1}{x^2}$.

   Solution: The limit has indeterminate form 0/0, so we apply l'Hospital's rule to get
$$\lim_{x\to 0} \frac{\cos x - 1}{x^2} = \lim_{x\to 0} \frac{-\sin x}{2x}.$$
   This again has the indeterminate form 0/0, so we apply l'Hospital's rule again to get
$$\begin{aligned} \lim_{x\to 0} \frac{-\sin x}{2x} &= \lim_{x\to 0} \frac{-\cos x}{2} \\ &= \boxed{-\frac{1}{2}.} \end{aligned}$$

2. Find $\lim_{x\to 0} \left(1+\frac{x}{3}\right)^{\frac{1}{x}}$.

   Solution: This limit has the indeterminate form 1^∞. Let $y = \lim_{x\to 0} \left(1+\frac{x}{3}\right)^{\frac{1}{x}}$. Then we have
$$\begin{aligned} \ln y &= \ln \lim_{x\to 0} \left(1+\frac{x}{3}\right)^{\frac{1}{x}} \\ &= \lim_{x\to 0} \ln \left(1+\frac{x}{3}\right)^{\frac{1}{x}} \\ &= \lim_{x\to 0} \frac{\ln\left(1+\frac{x}{3}\right)}{x}. \end{aligned}$$
   This last limit has the indeterminate form 0/0, so we apply l'Hospital's rule to get
$$\begin{aligned} \ln y &= \lim_{x\to 0} \left(\frac{1}{1+\frac{x}{3}}\right) \cdot \frac{1}{3} \\ &= \frac{1}{3}. \end{aligned}$$
   Thus $\boxed{y = e^{\frac{1}{3}}.}$

3. Compute $\int \frac{x}{\sqrt{1-x^2}}\,dx$.

   Solution: Let $u = 1 - x^2$ so that $du = -2x\,dx$ and thus $dx = -\frac{du}{2}$. Making this substitution, the integral becomes
$$\begin{aligned} -\frac{1}{2}\int \frac{du}{\sqrt{u}} &= -\frac{1}{2}\left[2u^{\frac{1}{2}}\right] + C \\ &= -\sqrt{u} + C \\ &= \boxed{-\sqrt{1-x^2} + C.} \end{aligned}$$