Math 202(01) Solution
Quiz 3 15 February 2002

1. Compute $\int x \ln x \, dx$.

   Solution: We use integration by parts, with $u = \ln x$, $du = \frac{dx}{x}$, $dv = x\,dx$, and $v = \frac{x^2}{2}$. We get

$$\begin{aligned}\int x \ln x \, dx &= \frac{x^2 \ln x}{2} - \int \frac{x}{2}\, dx \\ &= \boxed{\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C.}\end{aligned}$$

2. Compute $\int \tan x \sec^4 x \, dx$.

   Solution: We have

$$\int \tan x \sec^2 x \sec^2 x \, dx = \int \tan x (\tan^2 x + 1) \sec^2 x \, dx.$$

   We let $u = \tan x$ and substitute to get

$$\begin{aligned}\int u(u^2+1)\, du &= \frac{u^4}{4} + \frac{u^2}{2} + C \\ &= \boxed{\frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + C.}\end{aligned}$$

   Alternatively, we can write

$$\int \sec^3 x \sec x \tan x \, dx$$

   and let $u = \sec x$ to get

$$\begin{aligned}\int u^3 \, du &= \frac{u^4}{4} + c \\ &= \boxed{\frac{\sec^4 x}{4} + C.}\end{aligned}$$

3. Compute $\int_0^\pi \sin^2 x \, dx$.

   Solution: We have

$$\begin{aligned}\int_0^\pi \sin^2 x \, dx &= \frac{1}{2} \int_0^\pi 1 - \cos 2x \, dx \\ &= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right]_0^\pi \\ &= \frac{1}{2} [(\pi - 0) - 0] \\ &= \boxed{\frac{\pi}{2}.}\end{aligned}$$