Math 202(01) Solution
Quiz 4 1 March 2002

1. Write out the form of the partial fraction decomposition for $\dfrac{1}{x^4-x^2}$. Do not try to evaluate the coefficients.

   Solution: The denominator factors as
$$x^4 - x^2 = x^2(x^2-1) = x^2(x-1)(x+1)$$
   so the correct form for the partial fractions decomposition is
$$\frac{1}{x^4-x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}.$$

2. Compute $\displaystyle\int \frac{dx}{x(x-1)}$.

   Solution: We use partial fractions. We begin by writing
$$\begin{aligned}\frac{A}{x} + \frac{B}{x-1} &= \frac{1}{x(x-1)}\\ A(x-1) + Bx &= 1.\end{aligned}$$
   From which is follows that $A+B=0$ and $A=-1$, so $B=1$. We have
$$\begin{aligned}\int \frac{dx}{x(x-1)} &= \int -\frac{1}{x} + \frac{1}{x-1}\,dx\\ &= -\ln|x| + \ln|x-1| + C.\end{aligned}$$

3. Compute $\displaystyle\int \frac{x^2-3}{x+2}\,dx$.

   Solution: To begin, we divide $x+2$ into x^2-3. We get
$$\begin{array}{r|rrr} & x & - & 2 \\ \hline x + 2 & x^2 & & -\ 3 \\ & x^2 & +\ 2x & \\ \hline & & -\ 2x & -\ 3 \\ & & -\ 2x & -\ 4 \\ \hline & & & 1 \end{array}$$
   Thus
$$\begin{aligned}\int \frac{x^2-3}{x+2}\,dx &= \int x - 2 + \frac{1}{x+2}\,dx\\ &= \frac{x^2}{2} - 2x + \ln|x+2| + C.\end{aligned}$$