Math 202(01) Solution
Quiz 5 8 March 2002

Show all work. An answer without sufficient work shown may not receive full credit even if it is correct.

1. Use Simpson's rule with $n = 4$ to estimate $\int_1^3 \frac{1}{x}\,dx$. Round your answer to six decimal places, or leave it in the form of a fraction, or as a sum of five fractions.

   Solution: The division points are 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, and 3. We have

$$\begin{aligned}
S_4 &= \frac{1}{6}\left[f(1) + 4f\left(\frac{3}{2}\right) + 2f(2) + 4f\left(\frac{5}{2}\right) + f(3)\right] \\
&= \frac{1}{6}\left[1 + \frac{8}{3} + 1 + \frac{8}{5} + \frac{1}{3}\right] \\
&= \frac{1}{6} \cdot \frac{33}{5} \\
&= \frac{11}{10} \\
&= 1.1.
\end{aligned}$$

2. Use trapezoids with $n = 8$ to estimate $\int_1^2 \frac{1}{1+x^4}\,dx$. Round your answer to six decimal places.

   Solution: Let $f(x) = \frac{1}{1+x^4}$. Then we have

$$T_8 = \frac{1}{16}\left[f(1) + 2f\left(\frac{9}{8}\right) + 2f\left(\frac{10}{8}\right) + \cdots + 2f\left(\frac{15}{8}\right) + f(2)\right]$$

   We use a calculator to find that

$$f\left(\frac{9}{8}\right) + f\left(\frac{10}{8}\right) + \cdots + f\left(\frac{15}{8}\right) \approx 1.35510712626.$$

   We find that

$$f(1) + f(2) \approx 0.558823529412.$$

   Assembling our answers, we get

$$\begin{aligned}
T_8 &\approx 0.204314861371 \\
&\approx 0.204315.
\end{aligned}$$

3. Suppose we want to estimate $\int_1^3 \frac{dx}{x}$ with an error of less than $10^{-5}$, using Simpson's rule. What is the smallest value of $n$ we can use?

Solution: To find $K_4$, we write

$$f(x) = \frac{1}{x}, \quad f'(x) = -\frac{1}{x^2}, \quad f''(x) = \frac{2}{x^3}, \quad f'''(x) = -\frac{6}{x^4}, \quad f^{(\text{iv})}(x) = \frac{24}{x^5}.$$

The maximum value of $f^{(\text{iv})}(x)$ on $[1,3]$ is 24. Thus we have

$$\begin{aligned} E_S &\leq \frac{24(2)^5}{180n^4} \\ &= \frac{768}{180n^4}. \end{aligned}$$

We need to solve

$$\begin{aligned} \frac{768}{180n^4} &< 10^{-5} \\ \frac{10^5 \times 768}{180} &< n^4 \end{aligned}$$

so

$$\begin{aligned} n &> \sqrt[4]{\frac{10^5 \times 768}{180}} \\ &\approx 25.56. \end{aligned}$$

We take $n = 26$ to get the desired accuracy.